TEACHERS INVESTIGATING ADULT NUMERACY

Bundle #1

Contents

TERC

Number Sense: Flexibility & Fluency
Introduction

What is number sense?

Number Sense has been described as the common sense of numbers, a kind of mathematical literacy. A person with number sense knows what numbers mean and understands how to use them. According to Dr. Francis (Skip) Fennell, Co-Principal Investigator for the ***Numbers Alive!*** project, the primary and most consequential focus of number sense is on reasonableness: Is that answer or response reasonable? How do we know? If students are encouraged to think in those terms, and make connections between numbers and the information they observe in their lives everyday, they will develop a good sense of numbers. Few would argue that number sense, defined this way, is a useful and important adult basic skill, but little is known about how to help adults develop this sensibility.

This first TIAN Math Bundle focuses on the idea that people who have good number sense are flexible and fluent with numbers. By *flexible*, we mean they can solve problems and calculations in a variety of ways; by *fluent*, we mean they know when and how to use appropriate methods. They use computational strategies suited to the situation and purpose, including paper and pencil, calculators, mental math, and estimation.

The bundle begins with four activities that ask teachers to call upon and expand their own estimation abilities and mental math skills, and then to think of ways to integrate activities into their classrooms which purposefully provide opportunities for students to call upon and expand their estimation and mental math skills.

People who have number sense are good estimators.

They have a good sense when a calculation or computation is way off. For example, they know 5% tax on $6.49 just can't be $3.25. They have a good sense of "numerosity". They are good at strategically guessing how many jelly beans are in a jar, how many people are at the party, etc. They have a sense of measurements and know that they should allow about 3 hours to drive 150 miles on the highway because they average about 60 mph but need to figure in traffic and rest stops. They can eyeball a cup of sugar even without a measuring cup. An adult's ability to estimate most likely comes from real life experience. Someone who works in retail and calculates a 5% sales tax repeatedly has a sense of "5%". Good estimating is more than wild guessing. It is having a repertoire of "benchmarks" and experiences to draw upon and use flexibly. *Activity 1A. Personal Benchmarks* asks people to reflect on the bank of measurement and numerical benchmarks and referents they each have, and to expand that repertoire by sharing with one another.

Sowder (1992) wrote that there are three types of estimation: computational, numerosity, and measurement.

Computational estimation can be thought of as "finding an approximate answer to arithmetic problems without actually (or before) computing the exact answer" (Lemaire

& Lecacheur, 2002, p.282), for example, estimating what 13 cans of soup at three for $1.29 would be. There are many ways to estimate the answer. One way might be to think about 50 cents a can, so ten would be about $5, three more $1.29, so somewhere in the neighborhood of six dollars. How would you do it in your head?

Numerosity estimation is described as gauging the cardinal number of a discrete quantity (Brade, 2003), for example, guessing how many candies are in a jar, or how many names are on a page in a telephone book. *Activity 1B. About How Many?* provides some chances to use numerosity estimation.

Measurement estimation is a "sense of measures", such as judging how high the ceiling is by basing it on one's own height, or previous knowledge of how high ceilings normally are. *Activity 1C. The Million Dollar Activity*, challenges people to call upon measurement and computational estimation skills.

People who have number sense are good at mental math.

Being able to stand at the coffee shop counter in the morning and figure the price of two coffees and two muffins with tax in YOUR HEAD is a mark that you have some good number sense. Again, there are many ways to do that, but one cannot deny that knowing some number facts certainly help. In TIAN Institutes, we talked a lot about the value of seeking and identifying patterns as a way to become more fluent with numbers. *Activity 1D. Using Patterns to Strengthen Number Knowledge* provides one investigation, and we challenge TIAN teachers to think of other ways to make use of patterning.

So, we begin by offering four activities for teachers to do together and then to determine which might either go straight to the class or perhaps need some adaptation. More classroom resources to develop flexibility and fluency are suggested in *Classroom Resources* and some articles for your personal research and further readings on the topic are listed in *Articles & References*. We also ask that TIAN teachers reflect about how this topic—Number Sense: Flexibility and Fluency—is reflected in your *State Frameworks*.

Whether you are a member of a local or regional group, or working on your own this year, please share additional resources and your thoughts on TIAN Bundle #1 with your TIAN colleagues in Arizona, Kansas, Louisiana, Massachusetts, Ohio, and Rhode Island via the TIAN Talk discussion list. As a TIAN participant or group facilitator, you will automatically be subscribed to the TIAN Talk discussion list. To post a message to this list, send an email to tian-talk@cls.coe.utk.edu. To view the list archives or to manage your email subscription, go to http://cls.coe.utk.edu/mailman/listinfo/tian-talk.

Have Fun!

Brade, G.A. (2003). *The effect of a computer activity on young children's development of numersoity estimation skills.* Unpublished doctoral dissertation. University of New York at Buffalo.

LeMaire, P. & Lechacheur, M. (2002). Children's strategies in computational estimation. *Journal of Experimental Child Psychology, 82*, 281-304.

Sowder, J. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 371-389). New York: MacMillan

Number Sense: Flexibility & Fluency

Activity 1A. Personal Benchmarks

Goal: To explore and share personal numerical and measurement benchmarks that people use when estimating

Time estimate:

30–40 minutes

Focus:

Number Sense: Flexibility & Fluency
Estimation, Measurement

Materials:

✓ Handout *My Personal Benchmarks* (page 5)

Preparation

Gather a variety of materials, including objects of different weights and lengths.

This activity is designed for teachers first to do together, and then to bring back to their classes to share as is or adapt for their classrooms. PLEASE SHARE INTERESTING PERSONAL BENCHMARKS THAT ARISE IN TEACHER or STUDENT DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

5 m. **1.** Ask participants to think about a time when they had to estimate something (length, weight, etc.) but they did not have any measurement tools or a sense of the size of the amount. You might begin with your own example if participants are stuck trying to offer suggestions (for example, the width of the fingernail on the pinkie is about 1 cm on my hand, or I use my foot as 1' to measure a room's dimensions). Have volunteers share a few examples just to get others thinking about the topic of measurement benchmarks.

Also ask people to think of useful numerical benchmarks. For example, 10% is useful because finding 10% of a number is pretty easy to do, so it can help estimating other percent portions of an amount.

10 m. **2.** Distribute the *My Personal Benchmarks Handout* and ask participants to complete it as best they can within the time allotted. This part of the activity should not take more than 5–10 minutes. Notice how individuals are doing. When everyone has at least one or two examples jotted down, move on to the next step.

10 m. **3.** Now ask individuals to turn to a partner and take turns sharing their examples. Be sure to give them time and space to actually act out some of the examples, if they want. For example, if someone says that his foot is about 12 inches in length, another individual may want to consider what his estimated foot length is as well. This encourages participants to continue to develop their own personal benchmarks.

10 m. **4.** Bring the entire group back together. Ask

- What did you learn about how people estimate?
- How useful do you think it is to have personal benchmarks?
- How might you adapt this activity for your class?

For facilitators/teachers to think about

Remember to LISTEN and OBSERVE at the beginning.

The purpose of this activity is to unveil people's personal benchmarks, so don't go ahead and teach—"Just use your foot for a foot..."

However, if you discover in your class that few people have a sense of how long a foot, an inch, or a yard are, or how to estimate with benchmarks numbers such as ½, ¼, and 1/10, it will give you ideas for some future lessons.

My Personal Benchmarks Handout

Think about a time when you had to estimate something. Maybe you didn't have the right tool handy. Or, maybe you were trying to figure out just how big a large amount really was, or how much money something would cost.

Jot down your examples of "benchmarks" or points of reference in the first column. Then think about where you make use of that estimation and complete the other columns, if possible.

Measurement or Numerical Benchmark	What it represents	Where have you used this benchmark in your life?	What are some other places where you might use this benchmark?
Example: My FOOT	It is about 1'	I use it to estimate the dimensions of a room.	Helps me see how far I am parked from a hydrant.
Example: One drop of ink in an Olympic swimming pool	The idea of 1 ppb One part per billion	I used it to understand air quality reports measuring toxins.	Quality issues in the workplace

Number Sense: Flexibility & Fluency

Activity 1B. About How Many?

Goal: To explore strategies for estimating numerosities (amounts of items) in different situations

Time estimate:

45 minutes

Focus:

Number Sense: Flexibility & Fluency
Estimating Numerosities

Materials:

- ✓ Handout "About How Many"
- ✓ Jar of candy, marbles, beans, or similar objects, where you know the count
- ✓ Photo of an ear of corn (p. 9)
- ✓ Photo of cars in a parking lot (p. 10)
- ✓ Clear plastic bag of rice (optional)
- ✓ Marker and large piece of chart paper

Preparation

Large piece of chart paper with the items to be estimated written across the top [for step 4, you will use this to capture the various estimates that people came up with].

If doing this with a large group, you may want to create three stations—one with the jar of marbles, one with the ear of corn photo, and one with the car photo.

This activity is designed for teachers to first do together, and then to bring back to their classes to share as is or adapt for their classrooms. PLEASE SHARE INTERESTING STRATEGIES THAT ARISE IN TEACHER or STUDENT DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

5 m. **1.** Ask participants to think about a time when they had to estimate something that seemed too difficult to count individually. Ask: *What strategies did you use to estimate the total?* Have volunteers share their ideas.

15 m. **2.** Distribute the *About How Many?* Handout with photos and one jar of objects of which you know the count and tell participants to roughly estimate the total for each item. Next, they are to work in pairs to revise their estimate based on their own and their partner's strategies and complete the worksheet.

10 m. **3.** Now ask pairs to turn to another pair and take turns sharing their estimates and their strategies. Together, the group of four should reach a consensus and be prepared to support their reasoning.

15 m. **4.** Bring the entire group back together. Ask group reps to record their estimates on the large piece of chart paper. If there are large discrepancies in number, discuss various strategies that individuals used to see whether the type of strategy used influenced the size of the estimate.

Then ask:

- What did you learn about how people estimate?
- How do the strategies change as the situations change?
- How might you adapt this activity for your class?

For facilitators/teachers to think about

Pairs are asked to figure out estimation strategies and then to reach a consensus with another pair. This was written to get some collaborative problem-solving going. Does that happen well in your group? If not, how might you restructure the sharing of strategies?

How well do people deal with the uncertainty that no one really knows (nor can or want to prove) how many corn kernels or grains of rice there are precisely? Certainly, the person who fills the candy jar knows the precise answer, and the car picture can be counted ahead of time, but not so for the corn and rice.

About How Many? Handout

You will estimate the total number of objects in three different situations. First, do a very rough estimate. Then use your own strategy to refine your estimate. In the last column, explain your strategy.

Item to be estimated	Initial rough estimate	Revised estimate based on a strategy	Strategy used to refine estimate
Total number of kernels on the ear of corn			
Number of objects in jar			
Number of cars shown in the parking lot			

Number Sense: Flexibility & Fluency
Activity 1C. The Million Dollar Activity

Goal 1: To use strategies for measurement estimation and computational estimation to make a decision about relative values.

Goal 2: To think about how to "adultify" a math-rich activity written for 6th graders for an adult ed classroom.

This activity comes from an article in the NCTM journal, *Mathematics Teaching in the Middle School* which can be a great resource for ABE classroom activities. To read about how the activity played out in a sixth grade classroom in Syracuse, NY, click here.

Time estimate:

90 minutes
(45 minutes for the problem plus 45 minutes for "adultification")

Focus:

Number Sense: Flexibility & Fluency
Estimation, Measurement

Materials:

- ✓ Handout 1 "The Million Dollar Activity"
- ✓ The article with information about the Carrier Dome.

For each group:
- ✓ One dollar bill
- ✓ Calculator
- ✓ String, between one yard and two yards in length

For the class
- ✓ A stack of 50 $1 bills or a photo of 50 $1 bills with a ruler next to it
- ✓ Handout 2 "The Million Dollar Activity (for Adult Learners)"
- ✓ "The Million Dollar Activity" article

Preparation

Try the activity yourself!
Read the NCTM article, "The Million Dollar Activity."
Enlarge the above photo of the stack of dollar bills by 200%, if you don't have one.

This activity is designed for teachers first to do together and then to bring back to their classes to share as is or adapt for their classrooms. PLEASE SHARE INTERESTING STRATEGIES OR WAYS THAT YOU ADAPTED THE ACTIVITY FOR THE CLASSROOM ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence—Part 1:

5 m. **1.** Ask participants to think of the largest amount of cash they have ever seen. How much money was it? About how much room did it take up? Bring up issues of denomination. A million dollars in pennies takes up a lot more room than a million dollars in $100 bills. In this activity, we will think about how much space 1,000,000 $1 bills take up.

30 m. **2.** Distribute "The Million Dollar Activity" handout, and the article on the history of the Carrier Dome. Form groups of 3 or 4. Ask each group to map out a plan of action and then decide which of four options in "The Million Dollar Activity" is best. They should be prepared to justify their answer.

10 m. **3.** Bring the groups together to share conclusions and strategies, and answers to questions 1–3.

- What benchmarks or previous knowledge did your group use to solve the problem?
- If you had to do a problem like this again in a group, what would you do differently?

For facilitators/teachers to think about

This is a complex problem and working in a group can sometimes add to the complexity. Encourage groups to agree on a plan before forging ahead. Perhaps different people can have different roles.

Suggested Activity Sequence—Part 2: Adultifying the Million-Dollar Activity (for teacher groups)

10 m. 1. Distribute the NCTM article, "The Million Dollar Activity" and present this scenario to your colleagues:

This problem was written to give 6th graders in Syracuse, NY, in a context that would grab them and as a math-rich problem that would give them a reason to work as a group to creatively use their estimation skills. The teachers wrote an article about some of the ways the 6th graders handled the problem. (Give everyone 5 minutes to read "Students Work on the MDA" on p. 429.)

Then ask

- Did you see your ways of mapping out a plan reflected there?
- What are the math ideas that get touched upon in this activity?

25 m. 2. Ask:

- Do you think you might try this problem as is in your class, or would you make some adjustments? What would they be?
- Can you think of other contexts that would be appropriate for your students around the same math ideas?

Brainstorm together in your small group and write one or two new handouts.

10 m. 3. At this point, people can pass the problems around, editing them for one another. Ask for a volunteer to type the problems up and send them to everyone. Share some on the TIAN-Talk discussion list at tian-talk@cls.coe.utk.edu.

Handout 1: The Million Dollar Activity

Which would you rather have?

1. A million dollars?
2. Syracuse's Carrier Dome football field covered with $1 bills goal line to goal line?
3. The football field covered with stacks of 50 $1 bills each goal line to goal line?
4. Dollar bills piled on top of one another in one tall skinny column from the center of the Carrier Dome field until it reaches the roof?

Report on your problem solving methods.

1. How did you use the dollar bill, the stack of 50 $1 bills, the ruler, and the string in your thinking about the problem?

2. Do you think your final answer is precise or an approximation? Why?

3. How sure are you of your answer? Why?

Handout 2: The Million Dollar Activity (for Adult Learners)

Written by: _____

Which would you rather have?

1. A million dollars?

2.

3.

4.

Materials Needed:

How To Facilitate:

Number Sense: Flexibility & Fluency

Activity 1D. Using Patterns

Goal: To think of ways to strengthen multiplication fact knowledge and to reinforce key properties that form the basis of algebra.

Time estimate:

35–40 minutes

Focus:

Number Sense: Flexibility & Fluency

Materials:

- ✓ Multiplication Table Handout (page 18)
- ✓ Overhead or large times table chart.

Preparation

Read the article *Integrating Arithmetic and Algebra*

This activity is designed for teachers to first do together, and then to bring back to their classes to share as is or adapt for their classrooms. PLEASE SHARE INTERESTING PATTERNS THAT ARISE IN THE TEACHER or STUDENT DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

10 m. **1.** Distribute the times table to everyone and ask if anyone uses something similar in their own classes, and if so, how. Then ask individuals to find as many patterns as they can. Encourage them to write those patterns in words and, if possible, algebraic generalizations.

5 m. **2.** Now ask individuals to turn to a partner and compare and share patterns they each found.

10 m. **3.** Bring the group together and ask volunteers to come to the front and show their patterns on the overhead or chart-size table. As people

share their patterns, push for the generalization by asking, *"Why is that?"*

10 m. **4.** Close by asking:

- How could you use this pattern-seeking activity with your students?
- Can you think of other ways to use pattern-seeking to help strengthen number facts?

For facilitators/teachers to think about

Any discussion on times tables can go on forever, so try to keep the focus on pattern recognition.

Be sure that the following key ideas are discussed during the debriefing:

➔ Multiplication of 0: any number x 0 = 0, or $0a = 0$. [This can be seen by looking down or across the 0's facts.]

➔ Identity property of 1: any number x 1 = that number, or $1b = b$. [This can be seen by looking down or across the 1's facts.]

➔ Property of 1: any number ÷ by itself = 1, or $b/b = 1$.

➔ Commutative property of multiplication: $ab = ba$. [This can be seen by looking at the mirror image around the diagonal going down to the right of the table.]

➔ Distributive property: $5(x - 2) = 5x - 10$. [Explore this using different numbers.] If n = row heading and b is one column heading and c is another column heading, then $n(b - c) = nb - nc$. In other words, the differences in the products is always n times the difference of the numbers. [This can be illustrated with the 9's rules: $n \times 9 = n(10 - 1) = 10n - 1n$.]

Multiplication Table Handout

X	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Write down all the patterns you see in the multiplication table.

Number Sense: Flexibility & Fluency

Connecting to State Standards

How and where do your state math standards (or benchmarks) address the topic of number sense: flexibility and fluency? Are estimation, mental math, and number relationships explicitly described? Implicitly assumed?

Below are links to each of the 6 states standards web pages.

Arizona

http://www.ade.az.gov/adult-ed/adult_ed_standards.asp

Kansas

http://adultnumeracy.terc.edu/pdfs/KS_state_standards.pdf

Louisiana

http://www.doa.louisiana.gov/osr/lac/28v129/28v129.doc.

Massachusetts

http://www.doe.mass.edu/acls/frameworks/

Ohio

http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&TopicRelationID=966&Content=21875

Rhode Island

http://www.brown.edu/lrri/standards.html,

Articles and References (For Teachers) About Strengthening Number Sense Flexibility and Fluency

Read and discuss some of these articles with fellow teachers. Use the attached Research Reading Response (p. 26) to help guide your discussions. Please share your thoughts with others by posting online to the TIAN Talk discussion list at tian-talk@cls.coe.utk.edu.

Suggestion: A good starting place would be to take Number Sense Quiz designed for teachers on pages 22–25, and then read the article by Menon.

Burns, Marilyn (2007). *Mental Math* from Instructor. (http://www.mathsolutions.com/documents/2007_Instr_MentalMath.pdf) Suggestions for teaching students to do math mentally.

Carroll, William M. (1996). *Mental Computation of Students in a Reform-based Mathematics Curriculum* (http://findarticles.com/p/articles/mi_qa3667/is_199610/ai_n8750956/pg_1). Describes results of mental math study with two classes of fifth graders.

Curry, Donna (Spring 2007). Integrating Arithmetic and Algebra, *The Math Practitioner*, 13 (1). (http://adultnumeracy.terc.edu/pdfs/integarithalg.pdf)

Menon, Ramakrishnan (Spring 2004). *Preservice Teachers' Number Sense* in Focus on Learning Problems in Mathematics. (http://findarticles.com/p/articles/mi_m0NVC/is_2_26/ai_n6154493) This article includes a ten-item assessment used with teachers to learn more about their own sense of numbers. Try it yourself!

Seeley, Cathy (2005). *Do the Math in Your Head* (http://www.nctm.org/uploadedFiles/About_NCTM/President/Past_Presidents_Messages/Seeley_Messages/2005_12_mathhead.pdf) Ex-president of NCTM talks about the need for mental math.

Ten Research Findings from "Adding It Up" (http://investigations.terc.edu/library/implementing/qa-1ed/10_findings.cfm). Short research findings based on the book *Adding It Up: Helping Children Learn* Math from the National Research Council.

What Is Computational Fluency? (http://users.ntplx.net/~region10/regiontenmathpages/region10mathsitefaq/whatiscomputatinalfluency.html) This brief piece provides an overview of NCTM's stance on computational fluency and flexibility.

What is Number Sense? (http://www.mpt.org/learningworks/teachers/numbers_alive/whatis.html). Current NCTM President defines number sense with concrete examples.

General Background Readings

Bransford, John; Brown, Ann; and Cocking, Rodney; eds. (1999). *How Experts Differ from Novices* (pp. 19–38) in How People Learn: Brain, Mind, Experience, and School. National Academy Press Washington, DC. Research notes how people move from novice to expert, including how flexibility and fluency help move people along the continuum.

To access this article, please click on the following link or cut and paste it into your browser window: http://www.nap.edu/catalog.php?record_id=9853

Below the book information and title is a box where you will see the “PDF options”. The second option is “PDF Chapters for $2”.

Hit “select”, then “agree” on the next page, and you will be taken to a Table of Contents for the book. Select *Chapter 2: How Experts Differ from Novices*, and then "add the chapter to cart". After this you can checkout.

A Number Sense Quiz for Teachers

Try these 10 items without a calculator. Then read the research study by Menon (http://findarticles.com/p/articles/mi_m0NVC/is_2_26/ai_n6154493) to see how a group of preservice teachers responded.

1. A taxicab is only allowed 4 passengers in one taxi. How many taxis would be needed to take 9 passengers? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. 2

   B. 2-1/4

   C. 3

   D. Not enough information given

   Explanation:

2. A cab driver picks up 4 passengers. The passenger's ages are 9, 11, 14 and 34. What is the age of the cab driver? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. 20

   B. 48

   C. 68

   D. Not enough information given

   Explanation:

3. About how many days old are you? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. 800

B. 8000

C. 80000

D. 800000

Explanation:

4. Given that 48 + 37 = 85, what is the answer to 49 + 36? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. 85

   B. 86

   C. 715

   D. 4126

   Explanation:

5. Given that 38 + 47 = 85, what is the answer to 85 - 47? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. 38

   B. 42

   C. 132

   D. 817

   Explanation:

6. How many different fractions are there between 2/5 and 3/5? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. None

   B. One

   C. A few

   D. Lots

   Explanation:

7. Which is the largest fraction? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. 4568/4569

   B. 4569/4570

   C. 499/500

   D. 500/501

   Explanation:

8. How many different decimal numbers are there between 1.52 and 1.53? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

   A. None

   B. One

   C. A few

   D. Lots

   Explanation:

9. Without calculating the exact answer, circle the best estimate for 292 X 0.96 Explain why you chose that answer. Draw pictures if you need to.

   A. slightly more than 292

   B. slightly less than 292

   C. 292

   D. Cannot tell without calculating it

   Explanation:

10. If a "broken" calculator displays 6858 as the answer to 15.24 X 4.5, where should you place the decimal point in the answer? Circle your answer. Explain why you chose that answer. Draw pictures if you need to.

    A. 6.858

    B. 68.58

    C. 685.8

    D. 0.6858

    Explanation:

Research Reading Response

Title/Author/Reference:

_____

_____

Main Ideas:

- What were the author's main ideas regarding adults/children *learning* mathematics?

- What were the author's main ideas regarding *teaching* mathematics to adults/children?

Applications:

- What are the implications for you as an ABE mathematics teacher?

Classroom Resources
To Strengthen Number Sense Flexibility and Fluency

This is a starter list of classroom resources that focus on building estimation and mental math abilities. If you know of other resources, please share with others by posting online to the TIAN Talk Discussion list at tian-talk@cls.coe.utk.edu.

Published and Online Resources

EMPower Materials

The EMPower books that focus on number sense weave frequent opportunities to build estimation strength throughout the lessons. Below are some locations where estimation and mental math skills are developed.

Everyday Number Sense: Mental Math and Visual Models includes "Mental Math Practice" in each of the 12 lessons. Lessons 1–6 particularly focus on mental math. Lesson 2, "Mental Math in the Checkout Line" is available in its entirety free online at the Key Curriculum Press site http://www.keypress.com/x18177.xml.

Using Benchmarks: Fractions, Decimals, and Percents, and *Split It Up: More Fractions, Decimals and Percents* intend to help students get a firm grounding with the benchmark fractions, ½, ¼, 1/10, 1/100, 1/3, and 1/8 and their decimal and percent equivalents. *Using Benchmarks,* Lesson 5, develops a sense of 1/10 and is available free online at http://www.keypress.com/x18176.xml. (Remember, it assumes Lesson 1–4.) *Split It Up*, Lesson 8, using nutrition labels as a context for percents, is also available free at http://www.keypress.com/x18173.xml. (This lesson is only suggested for use after covering lessons with content similar to Lessons 1-7.)

NCTM Illuminations Lessons

Illuminations is packed with activities and lessons for grades K–12, many of which can be adapted for adult learners. The lesson "Too Big or Too Small?" (http://illuminations.nctm.org/LessonDetail.aspx?id=L252), gives students an opportunity to explore large numbers and look at the reasonableness of answers.

Number and Operation: Authentic Learning Activities in Middle School Mathematics by Brendan Kelly, Brendan Kelly Publishing, Inc.

Wonderfully rich investigations using large numbers. "Mental Math Tips for Taxes and Tips" and "How High is the National Debt?" are examples. http://brendankellypublishing.com/

Mixing In Math

(http://mixinginmath.terc.edu/) offers a set of free online activities originally designed to provide math learning moments in after school situations (Grades K–7). The activities are quick and fun, and are easily adaptable to ABE. Two activities that establish time benchmarks are "How Many In a Minute?" and "Is It Possible?"

NOVA Online

The activity "How Many Pearls?" (http://www.pbs.org/wgbh/nova/pearl/uncountable.html) has students estimating the number of pearls in a chest using two strategies: weight and volume.

Sample Math WASL Questions

On this site (http://www.pasd.wednet.edu/school/mathWASL/), you can find a variety of questions for students to practice solving problems through written exploration. Questions are organized by topic (such as number sense, measurement, and geometry) by grade level and include answers. The Number Sense topic offers plenty of opportunities to become more flexible with numbers.

Ideas

Math Jeopardy

Begin by presenting a "solution" on the board, something like 24. Then ask students what the question to that answer might be. Examples of answers could include (but are not limited to) the following:

"How much is two dozen?"
"What is 3 x 8?"
"What is the square root of 144?"
"How many inches in two feet?"

Once students catch on to the idea, have them create answers for their peers. This can be a quick daily or weekly activity to stimulate flexibility in students' thinking about numbers.

Fill in the Blanks

Ask students to develop short stories in which the numbers have been replaced with blanks (depending on the level of students, you can place the numbers needed for the blanks at the top of the page or you can have them choose reasonable numbers out of their heads). Below is an example:

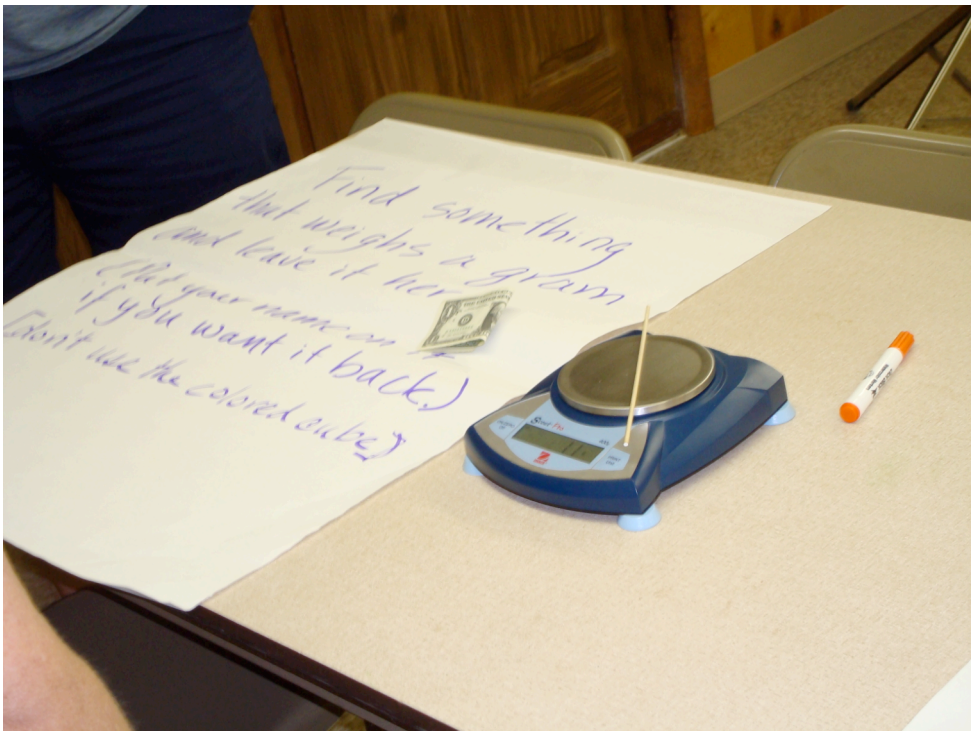
Valerie works _____ hours per week developing computer programs. During the weekend, she works another _____ hours landscaping. Between the two jobs, she earns about _____ yearly, which is just enough to pay _____ monthly for her apartment.

Hands-on Measuring

Provide opportunities to get "hands-on" with measurement units. Provide scales, rulers, measuring tapes, containers. Sample challenges to pose to the class:

- Find some things in the room that weigh a gram.
- Draw an inch.
- Draw a centimeter.
- Find some things that weigh about a pound.
- Find something that is about a yard long.

How tall is the palm tree? This fella starts out by using his own foot as one linear foot to gauge how long the tree's shadow is.



If you have a good scale, it's fun to find items that weigh about a gram.

Are We Really Teaching Numeracy?

Problem Statement

We are a group of teachers investigating adult numeracy (TIAN). But what is numeracy? Is it different from math? A paper was published recently that considers what numeracy includes. The authors examined a host of documents and concluded that numeracy is MORE than math, that it has three distinct yet intertwining components: context, math content, and cognitive/affective factors. Numeracy, defined this way, has not found its way into mainstream adult ed classrooms. Should it? If so, what would it take to make that happen? Read the excerpt below from this article and consider the accompanying questions with your regional or local group.

Suggested advance readings

"The Components of Numeracy" (especially the summary on p. 34)
http://www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf

The Adult Numeracy Network's "Teaching and Learning Principles" and "Professional Development Principles."
http://www.literacynet.org/ann/teachingandlearningprinciplesv610.30.05%20(newest).pdf

To gain insight into the importance of numeracy or quantitative literacy in today's society, select a few chapters that interest you from "Mathematics and Democracy: The Case for Quantitative Literacy."
http://www.maa.org/ql/mathanddemocracy.html

NCSALL Occasional Paper December 2006

Components and Subcomponents of Numeracy

CONTEXT – the use and purpose for which an adult takes on a task with mathematical demands

Family or Personal—as a parent, household manager, consumer, financial and health-care decision maker, and hobbyist

Workplace—as a worker able to perform tasks on the job and to be prepared to adapt to new employment demands

Further Learning—as one interested in the more formal aspects of mathematics necessary for further education or training

Community—as a citizen making interpretations of social situations with mathematical aspects such as the environment, crime and politics

CONTENT – the mathematical knowledge that is necessary for the tasks confronted

Number and Operation Sense—a sense of how numbers and operations work and how they relate to the world situations that they represent

Patterns, Functions and Algebra—an ability to analyze relationships and change among quantities, generalize and represent them in different ways, and develop solution methods based on the properties of numbers, operations and equations

Measurement and Shape—knowledge of the attributes of shapes, how to estimate and/or determine the measure of these attributes directly or indirectly, and how to reason spatially

Data, Statistics and Probability—the ability to describe populations, deal with uncertainty, assess claims, and make decisions thoughtfully

COGNITIVE AND AFFECTIVE—the processes that enable an individual to solve problems and, thereby, link the content and the context

Conceptual Understanding—an integrated and functional grasp of mathematical ideas

Adaptive Reasoning—the capacity to think logically about the relationships among concepts and situations

Strategic Competence—the ability to formulate mathematical problems, represent them, and solve them

Procedural Fluency—the ability to perform calculations efficiently and accurately by using paper and pencil procedures, mental mathematics, estimation techniques, and technological aids

Productive Disposition—the beliefs, attitudes, and emotions that contribute to a person's ability and willingness to engage, use, and persevere in mathematical thinking and learning or in activities with numeracy aspects

From Ginsburg, L., Manly, M., and Schmitt, M. J. (2006). *The components of numeracy* [NCSALL Occasional Paper]. Cambridge, MA: National Center for Study of Adult Literacy and Learning. Available: http://www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf

Questions to Guide Discussions

Discuss these questions in local or regional groups

1. According to the paper, numeracy has three components: context, content, and cognitive and affective factors. And each of those components has sub-components. How well does your teaching reflect those?

2. How well do your state mandated assessments and state standards reflect those components?

3. What do effective approaches to teaching numeracy look like?

4. Most tests that are used in adult education and for college placement focus mainly on skills. How does teaching numeracy with all its components prepare adults for such tests?

5. The Components of Numeracy paper suggests that numeracy is a complex proficiency that involves the intertwining of multiple skills and understandings. How do you recognize growth in something like conceptual understanding? In other words, what would a complete numeracy assessment look like?

6. For which of the numeracy components and subcomponents are we good at helping students make growth? Which are we not?

Introduction to the Facilitator's Guide

Each TIAN Bundle's third section (the Facilitator's Guide) is designed to give some practical suggestions about how to facilitate a teacher meeting using the resources in the other two bundle sections (Math Topic and Teaching/Learning Issue). Each Facilitator's Guide will also include some basic principles and good practices for leading a group. There is a suggested Meeting Feedback Form for the group and a Teacher Meeting Notes form to send to the tian-talk discussion list by sending an email to tian-talk@cls.coe.utk.edu. Please note these are only suggestions. The TIAN team is interested in hearing what groups decide is most important and helpful for them.

Suggestions for Using Bundle #1 in Teacher Meetings

As you plan to use a Bundle, print out a copy of the entire Bundle (about 40 pages). Read through it, deciding which sections to photocopy for the meeting and which to let group participants access themselves on the TIAN website at http://adultnumeracy.terc.eu/TIAN_teacher_resources.html

If your group has ONE two-hour meeting to spend on Bundle #1, set aside at least 2/3 of the time for the Math Topic and 1/3 of the time on the Teaching/Learning Issue. So a meeting might go something like:

1. **Introduce the Math Topic, Number Sense: Flexibility and Fluency, either by**
   a. summarizing the main points in the Introduction or
   b. emailing the introduction ahead of time to the group members, and then briefly discuss the main points in the meeting.
2. **Do some math together.**

   There are four activities. Choose two activities to do, for example: 1A Personal Benchmarks and 1C The Million Dollar Activity with time to "adultify" the activity.
3. **Consider the issue: "Are We Really Teaching Numeracy?"**

   Ask everyone to read the Problem Statement, the Components and Sub-components summary table, and select some of the questions for discussion.

4. **Get some feedback on the meeting and ask a volunteer to send an email to tian-talk to share good ideas that came up in the meeting. Also, ask everyone to bring back to the next meeting what they did with these activities in their classes.**

If your group has TWO two-hour meetings (4 hr) to use Bundle #1, you might spend the entire first meeting on the Math Topic, and the second meeting discussing how things played out in class, ending that second meeting with a discussion of the Teaching/Learning Issue.

In the first meeting, you might have time to do and reflect upon 3 activities, and to begin to chose some articles to read before the next meeting. You might start the second meeting with everyone taking the ten-item number sense assessment and read the Menon article later.

The Importance of Promoting Teacher Mathematical Learning

ABE math teacher groups get together for two main reasons—to get some good math teaching ideas and resources for their classrooms and to expand their own math knowledge. The activities that you do together begin with teachers wrestling with the problems themselves. As they struggle, some things you do as facilitator will be more likely to promote mathematical learning than others. All facilitators should keep these five important ideas in mind:

1. TIAN teachers value sharing solutions among themselves and encourage sharing in the classroom. When asking people to share, encourage people to explain their thought processes.
2. In the TIAN institutes we were always interested in more than one strategy, and whether we could see the connections between the strategies.
3. Regard confusion and error as learning opportunities—don't avoid it.
4. Raise honest questions that push on the math. This means it is ok to not have the answer to the questions posed. All of us are learners—that includes the facilitator.
5. It's a community—everyone should take responsibility for the learning.

These ideas, so beautifully presented in the table on the next page, would be good for everyone in the group to have a copy of right from the first meeting.

Continuum of Sociomathematical Norms

	Less likely to promote mathematical learning ⟷		More likely to promote mathematical learning
Sharing	Ideas and solutions are shared with minimal or no explanation	Thinking is described, often in procedural terms	Explanations consist of a mathematical argument
Solution Strategies	Emphasis is on one single solution or strategy	Multiple strategies and solutions are described	Emphasis is placed on the relationships among multiple solutions and/or strategies
Confusion & Error	Confusion and mistakes are avoided or ignored, or are corrected by the PD leader	Confusion and mistakes are acknowledged in hopes of causing disequilibrium and change in understanding	Confusion and errors are embraced as opportunities to compare ideas, re-conceptualize problems, explore contradictions in solutions, or pursue alternative strategies
Questioning	The PD leader asks questions aimed at maintaining social order or eliciting specific responses	Both the PD leader and teachers raise procedural and/or factual questions about the mathematics	Both the PD leader and teachers raise questions that push on understanding of mathematics/ mathematical reasoning
Community	Work is generally done individually or ideas are shared through PD leader explication	teachers collaborate to find solutions to problems	Mathematical argumentation forms the basis of a generative learning process where individuals take responsibility for their own and the group's progress

(Adapted from Yackel & Cobb)

Meeting Feedback Form

(for the group and the facilitator)

What was the most effective part of the meeting today, and why?

What would you change for the next time? Why?

What pressing issues/topics would be good to address?

Teacher Meeting Notes

(To share with other groups on the tian-talk discussion list at tian-talk@cls.coe.utk.edu)

Date/time of meeting:

Group Title and meeting location (City or town, State)

Facilitator(s)

Number of participants present

Describe what occurred at the meeting

Did you use any activities or discuss the issue from the TIAN Bundles? How effective were the activities or discussion of the issue?

Did your group use resources others than those in the TIAN Bundles? If so, please describe (or attach).