TIAN Bundles Articles and Reference for Teachers

TIAN Bundle 1 Articles and References about Strengthening Number Sense Flexibility and Fluency

- **Burns, M. (March/April 2007). Mental Math from *Instructor*.**
  Suggestions for teaching students to do math mentally.
- **Carroll, W.M. (1996). Mental Computation of Students in a Reform-based Mathematics Curriculum**
  Describes results of mental math study with two classes of fifth graders.
- **Curry, D. (Spring 2007). Integrating Arithmetic and Algebra. *The Math Practitioner*, Vol.13(1), p. 5-7.**
- **Menon, R. (Spring 2004). Preservice Teachers' Number Sense. *Focus on Learning Problems in Mathematics, Vol. 26(2).***
  This article includes a ten-item assessment used with teachers to learn more about their own sense of numbers. Try it yourself!
- **Seeley, C. (2005). Do the Math in Your Head**
  Ex-president of NCTM talks about the need for mental math.
- **Ten Research Findings from "Adding It Up"**
  Short research findings based on the book Adding It Up: Helping Children Learn Math from the National Research Council.
- **Fennell, F. (2008). Number Sense - Right Now! from *NCTM News Bulletin***
  Former NCTM President defines number sense with concrete examples.

General Background Readings

- **Bransford, J., Brown, A. & Cocking, R. (Eds.) (1999). How Experts Differ from Novices (pp. 19-38) in *How People Learn: Brain, Mind, Experience, and School*. National Academy Press Washington, DC.**
  Research notes how people move from novice to expert, including how flexibility and fluency help move people along the continuum. To access this article, go to http://www.nap.edu/catalog.php?record_id=9853. Below the book information are Option tabs. Choose the second option "Contents", and you will be taken to a Table of Contents for the book. Select Chapter 2: How Experts Differ from Novices.

TIAN Bundle 2 Articles and References about Strengthening Operation and Symbol Sense

About Operation Sense

- **Gregg, J. & Underwood, Diana. (May 2007). Measurement and Fair-Sharing Models for Dividing Fractions. *Mathematics Teaching in the Middle School*, 12(9), 490-496.**
  Includes examples of how to move students from division of whole numbers to division of fractions.
- **de Silva Lamberg, T. (September 2007). Student Approaches to Unitizing in Fair-Share Problems. *Mathematics Teaching in the Middle School*, 13(2),114-116.**
  This short article analyzes four students' methods for dividing (fair-share situation).
- **Nugent, P. (September 2007). Lattice Multiplication in a Preservice Classroom. *Mathematics Teaching in the Middle School*, 13(2),110-113.**
  Lattice multiplication is an alternative algorithm for teaching multiplication that helps students see the place values.
- **Taber, S. (December 2006). Using Alice in Wonderland to Teach Multiplication of Fractions. *Mathematics Teaching in the Middle School*, 12(5), 244-249.**
  Although practitioners won't be reading *Alice in Wonderland* to their students, this article does provide examples of the difference between proportional and nonproportional thinking with fractions.
- **Tent, M. (August 2006). Understanding the Properties of Arithmetic: A Prerequisite of Arithmetic. *Mathematics Teaching in the Middle School*, 12(1), 22-25.**
  Provides strategies for teaching the commutative, associative, distributive, and identity properties of addition and multiplication.

About the Understanding of the Equal Sign

- **Falkner, K., Levi, L., & Carpenter, T. (1999). Children's Understanding of Equality: A Foundation for Algebra.**
  This brief article describes the misconceptions students develop at a very early age about the equal sign. Included are strategies one teacher used to try to overcome those misconceptions.
- **Freiman, V. & Lee, L. (2004). "Tracking Primary Students' Understanding of the Equality Sign". Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (vol.2, pp. 415-422).**
  This research was aimed at tracking young students' understanding of the equal sign by monitoring how they responded to simple problem types such as $a + b = c$ and $c = a + b$. Types of errors in children in K, grade 3, and grade 6 were then analyzed.
- **Knuth, E. & Stephens, A. (July 2006). Does Understanding the Equal Sign Matter? Evidence from Solving Equations. *Research in Mathematics Education*, 37(4), 297-312.**
  This research project investigated the relationship between being able to solve simple algebraic equations and an understanding of the equal sign.

Other Related Articles

- **Algebraic Thinking from Assessment Resource Banks: English, Mathematics, and Science. https://arbs.nzcer.org.nz/conceptual-maps - algebraic-thinking-concept-map**
  This collection of mini-resources includes hands-on strategies for teaching equality and the additive, commutative, and associative properties, beginning very early on in learners' math education.
- **Computation, Calculators, and Common Sense (May 2005).**
  A position paper from the National Council of Teachers of Mathematics in reaction to the question of calculator use in the classroom.
- **National Research Council (2001). "Number: What Is There to Know?" from *Adding It Up: Helping Children Learn* (71-114).**
- **Schifter, D., Bastable, V., Russell, S.J., Riddle, M. & Seyferth, L. (2008). Algebra in the K-5 Classroom: Learning Opportunities for Students and Teachers. In Greenes, C. E. and Rubenstein, R. (Eds.), *Algebra and Algebraic Thinking in School Mathematics: 70th Yearbook* (pp. 263-277). Reston, VA: NCTM.**
  Discussion of classroom investigations into how students develop algebraic thinking.
- **Stiff, L. (April 2001). Making Calculator Use Add Up from *NCTM News Bulletin 37*, Vol. 3**
  A response to a newspaper article about the use of calculators in the classroom.

TIAN Bundle 3 Articles and References about Positive and Negative Integers

- **Ask Dr. Math FAQ: Negative x Negative = Positive from The Math Forum @ Drexel.**
  Provides several examples of rationale for why multiplying two negatives yields a positive integer.
- **Baldwin, J. & Kessel, C. (2000). Concept and Computation: The Role of Curriculum from *MER Newsletter*.**
  A look at how and when Chinese and American curricula handle negative numbers.
- **Ball, D.L. (1993). With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics**
  Based on an earlier version presented at the April 1990 meeting of the American Educational Research Association in Boston. While the dilemmas Ball discusses are about issues much larger than the topic of negative numbers, pages 7-17 and pages 23-31 of the article focus specifically on examples in teaching negative numbers.
- **Gregg, J. & Gregg, D. (August 2007). A Context for Integer Computation from *Mathematics Teaching in the Middle School*, 13(1), 46-50.**
  Suggests the context of allowance rather than debt or other "contrived" situations for bringing more meaning to integer computation.
- **Kent, L.B. (September 2000). Connecting Integers to Meaningful Contexts in *Mathematics Teaching in the Middle School*, 6(1), 62-66.**
  This article illustrates how students can begin to intuitively learn the rules for adding and subtracting integers.
- **Felder, K. (1997). Negative Times Negative Is What?**
  This site offers four explanations to the question of why multiplying two negatives gives you a positive number.
- **National Research Council (2001). "Number: What Is There to Know?" from *Adding It Up: Helping Children Learn* (80-83, 244-246).**
- **Nurnberger-Haag, J. (September 2007). Integers Made Easy: Just Walk It Off in *Mathematics Teaching in the Middle School*, 13(2), 118-121.**
  This quick read offers a multisensory method to teach the rules associated with adding, subtracting, multiplying and dividing integers.
- **Peled, I., & Carraher, D. (2008). Signed numbers and algebraic thinking. In D.W. Carraher & M.L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 303-328). New York: NCTM.**
  Researchers investigate the relationship between algebra and signed numbers and suggest that algebra provides more meaningful models for signed numbers.
- **Sfard, A. (2008). Learning Mathematics as Developing a Discourse. In R. Speiser, C. Maher, C. Walter (Eds), *Proceedings of 21st Conference of PME-NA* (pp. 23-44). Columbus, Ohio: Clearing House for science, mathematics, and Environmental Education.**
  While Sfard's key message is the importance of communication in learning mathematics, one of her two examples focuses on students grappling with negative numbers.

TIAN Bundle 4 Articles and References about Algebraic Thinking

- **Driscoll, M. & Moyer, J. (January 2001). Using Students' Work as a Lens on Algebraic Thinking, *Mathematics Teaching in the Middle School*, 6(5), 282-287.**
  This article focuses on the use of student work to help teachers focus on teaching and learning algebra.
- **Kalchman, M.S. (August 2005). Walking through Space: A New Approach for Teaching Functions in *Mathematics Teaching in the Middle School*, 11(1), 12-17.**
  Kalchman shares a real-life scenario for introducing several key aspects of linear functions.
- **Kriegler, S. Just What is Algebraic Thinking?**
  Kriegler discusses three lenses for looking at algebra: algebra as abstract arithmetic, algebra as language, and algebra as a tool for the study of functions and mathematical modeling.
- **Mooney, E.S. (March 2007). The Thinking of Students: Cookies, *Mathematics Teaching in the Middle School*, 12(7), 374-377.**
  This brief article gives insight into students' mathematical thinking about this problem: *Tim ate 100 cookies in 5 days. Each day he ate 6 more than the day before. How many cookies did he eat on the first day?*
- **Mooney, E.S. (December 2006). The Thinking of Students: Elizabeth's Long Walk, *Mathematics Teaching in the Middle School*, 12(5), 63-265.**
  This brief article gives insight into students' mathematical thinking about this problem: *Elizabeth visits her friend Andrew and then returns home by the same route. She always walks 2 km/h when going uphill, 6 km/h when going downhill, and 3 km/h when on level ground. If her total walking time is 6 hours, then what is the total distance she walks?*
- **Peterson, B.E. (November 2006). Counting Dots and Measuring Area: Rich Problems from Japan, *Mathematics Teaching in the Middle School*, 12(4), 214-219.**
  Students look at a set of dots and create a variety of generalizable patterns.
- **Smith, M.S., Hillen, A.F. & Catania, C.L. (August 2007). Using Pattern Tasks to Develop Mathematical Understandings and Set Classroom Norms, *Mathematics Teaching in the Middle School*, 13(1), 38-44.**
  This article discusses the use of pattern blocks to help students develop algebraic reasoning and to establish classroom norms and practices.
- **Thomas, D.A. & Thomas, R.A. (October 1999). Discovery Algebra: Graphing Linear Equations in *The Mathematics Teacher*, 92(7), 569-572.**
  In this article, Thomas shares his personal classroom experience moving toward a new approach to teaching algebra.

Additional Resources

Driscoll, Mark. (1999). *Fostering Algebraic Thinking: A Guide for Teachers*, Grades 6-10. Portsmouth NH: Heinemann.

TIAN Bundle 5 Articles and References about Geometric Thinking

About van Hiele:

- **Malloy, C. (2002). The van Hiele Framework.**
  Provides an explanation of van Hiele's model.
- **Mason, M. (n.d.) Van Hiele Levels of Geometric Reasoning.**
  Provides some concrete examples of various levels of the van Hiele model.
- **Woleck, K.R. (2003). Tricky Triangles: A Tale of One, Two, Three Researchers in *Teaching Children Mathematics*, (10)1, 40-44.**
  While the researchers focus on first grade, the article illustrates the sort of thinking at van Hiele's lowest level of geometry.

About Triangles:

- **Maxwell, S.A. (2006). Measuring Tremendous Trees: Discovery in Action in *Mathematics Teaching in the Middle School*, (12)3, 132-139.**
  The "shadow" problem comes to life in this article about measuring the height of a tree using a handmade clinometer.
- **Reynolds, M.J. (2002). Letting the Cat out of the Bag….to Make Room for a Triangle in *Mathematics Teacher*, (95)1, 6-7.**
  An interesting activity to introduce the idea of congruence — having groups of students figure out the shape of a triangle in a bag.

About Perimeter and Area:

- **Beck, S.A., Huse, V.E., & Reed, B.R. (2007). How Does Your Mathematical Garden Grow? in *Mathematics Teaching in the Middle School*, (13)2, 69-76.**
  In this article, middle school students move from determining perimeter and area to discovering functional relationships with perimeters and areas.
- **Pagni, D.L. (2006). Finding Areas on Dot Paper in *Mathematics Teaching in the Middle School*, (12)5, 274-278.**
  Students using dot paper to find the area of rectangles, triangles, parallelograms, and trapezoids readily see how the various formulas were derived. Sample worksheets are included.

Other:

- **Cipoletti, B., & Wilson, N. (2004). Turning Origami into the Language of Mathematics in *Mathematics Teaching in the Middle School*, (10)1, 26-31.**
  Explains how to effectively use origami in the classroom to teach geometric vocabulary.
- **Roberts, S. (2007). On My Mind: Not All Manipulatives and Models Are Created Equal in *Mathematics Teaching in the Middle School*, (13)1, 6-9.**
  Something to think about as you use readily-available manipulatives in your class.
- **Tepper, A.B. (1999). A Journey through Geometry: Designing a City Park in *Teaching Children Mathematics*, 348-352.**
  The description of an integrated curriculum project that would be applicable to students of all ages.
- **Morris, B. (2004). The Beauty of Geometry in *Mathematics Teaching in the Middle School*, (9)7, 358-361.**
  A hands-on project that allows students to be creative while learning the vocabulary of geometry.