Teachers Investigating Adult Numeracy

Bundle #5
Geometric Thinking

Contents

Geometric Thinking
Introduction

All of us live in a three-dimensional world full of angle, shape, and dimension. How can we use that world—rich with geometry—in our classrooms to shore up more formal understanding of geometric relationships? How do we build on adults' wealth of informal knowledge and move beyond it?

The first step may be to get everyone noticing, acknowledging, and discussing the geometry they experience.

- ❖ Activity 5A—*Triangles in Real Life* launches an exploration of the triangles in bridges, church steeples, see-saws, apple turnovers, etc., from photos found on Google Images.
- ❖ Activity 5B—*Larger than Life* starts out with an assortment of rectangular objects found around the home or work.
- ❖ Gardens are the milieu for Activity 5C—*Don't Fence Me In!*
- ❖ Quilting offers a rich source for geometric explorations in Activity 5D—*Geometry and Quilting*

The second step then is to connect the authentic experiences and materials of real life with geometric ideas.

- ❖ Activity 5A: *Triangles in Real Life* deals with the **properties of the triangle** and some patterns and relationships between angles and sides.
- ❖ Activity 5B: *Larger than Life* develops the idea of **similarity and scale**, as well as the relationship between dimensions, perimeter, and area of similar shapes.
- ❖ Activity 5C: *Don't Fence Me In!* pushes on looking for patterns and informal rules for **perimeter and area**.
- ❖ Activity 5D: *Geometry and Quilting* focuses on **symmetry and transformations**.

All along the way, teachers need to think seriously about how geometric thinking develops and how instruction can support that development. While there is no research we know of that has tried to understand how adults develop geometric thinking and spatial sense, we believe we can learn a lot from theories about how children's and adolescents' thinking develops. One theory offered by Pierre and Dina van Hiele, two Dutch educators, is particularly useful.

The van Hieles developed a five-step model for geometric thinking after noticing that many secondary school students were not able to grasp the more difficult geometric concepts taught. The model is based on how people learn as they gain insights and attain new levels of understanding. The model has been researched since it was first developed in the 1950s, and has continued to gain international acceptance as a way for describing stages of geometric understanding.

According to the Van Hiele theory, students develop their understanding of geometry figures by progressing through a series of five levels.[1]

Level	Description
1: Visualization, Recognition, or Concrete level	Students can name common geometric figures based on their shapes but not their characteristics. *This is a rectangle because it has four sides.*
2: Analysis or Descriptive level	Students can sort geometric figures based on their attributes. *The area of this triangle is the space inside; the perimeter is the length all around the triangle.*
3: Informal Deduction, Relational, or Ordering level	Students can classify figures and can accurately describe them. They discover and formulate generalizations and then develop informal arguments to support these generalizations. *I can figure out the area of this rectangle by counting all the square units within it; or I can multiply one side by the other. I can use different numbers and it still works.*
4: Deduction level	Students can perform geometric proofs and understand axioms and theorems. *This is a topic often covered in high school math, but rarely covered in adult education classes.*
5: Rigor level	Students are ready for non-Euclidean geometries. *This is an area reserved for higher level math classes.*

The van Hieles used this model to help determine the types of instruction that need to happen in order for students to develop into geometric thinkers. Van Hiele levels are sequential and hierarchical. Instruction, not age, helps students move from one level to another. Instruction must be at the appropriate level; students cannot skip levels.

Understanding this progression can help teachers consider how to provide more opportunities for their students to develop a stronger foundation of geometric reasoning by allowing students to build from where they are now. Teachers need to give students more time to explore geometric shapes, including their attributes so that they can discover their own rules and generalizations. From these rules, students can then naturally discover formulas. This discovery is so important to developing a strong sense of geometric reasoning, much more meaningful than memorizing formulas without understanding where they come from.

With our adult education students, we need to be sure that we are not trying to teach concepts at a level out of their reach. As teachers, we need to make sure that students can define what makes a triangle a triangle and not a rectangle, what makes a circle a circle, etc. We need to be sure they can do this well before we ask them to solve geometry problems similar to those on the GED test, college placement tests, or other standardized assessments. The activities in this Bundle are intended to provide students

[1] Sometimes the five levels are numbered 1-5, other times 0-4, depending on the author.

(and teacher participants) with opportunities to explore geometric thinking within levels 1 – 3.

More classroom resources to develop geometric thinking are suggested in Classroom Resources and some articles for your personal research and further readings on the topic are listed in Articles & References. We also ask that TIAN teachers reflect about how this topic—Geometric Thinking—is reflected in your State Frameworks.

Whether you are member of a local or regional group, or working on your own this year, please share additional resources and your thoughts on Math Topic #5 with your TIAN colleagues in Arizona, Kansas, Louisiana, Massachusetts, Ohio, and Rhode Island via the TIAN Talk discussion list. As a TIAN participant, you will automatically be subscribed to the TIAN Talk discussion list. To post a message to this list, send an email to tian-talk@cls.coe.utk.edu. To view the list archives or to manage your email subscription, go to http://cls.coe.utk.edu/mailman/listinfo/tian-talk.

Have Fun!

Geometric Thinking
Activity 5A. Triangles in Real Life

The following set of activities was developed by Sarah Fearnow, a TIAN teacher from Pima College in Tuscon, Arizona. The TIAN staff thanks Sarah for her willingness to share the lesson with other TIAN teachers.

Goal: To discover patterns of equilateral, scalene, and isosceles triangles

Time estimate:

1 hour

Focus:

Geometric Thinking

Materials:

- ✓ *Triangle Mind Map* Handout, p. 7
- ✓ *Triangle Research* Handout, p. 8
- ✓ Rulers
- ✓ Protractors
- ✓ Photo images (see below)

Preparation

Gather a variety of photo images from the Internet of things with equilateral, scalene, and isosceles triangles or triangular shapes. The images should be from real life, for example, bridges, church steeples, the Bermuda triangle, sculpture. Google Image Search is a great resource. http://images.google.com/.

Create a chart on the board or flip chart with three columns: student name, number of equal sides, and number of equal angles.

This activity is designed for teachers to do together, and then to think about the implications for their classrooms. PLEASE SHARE INTERESTING COMMENTS THAT ARISE IN TEACHER DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

This is a lesson that I used with my GED math class at Pima College Adult Education. The ideas for the lesson were inspired by TIAN. We did this lesson in a unit on Geometry, after having studied angles.

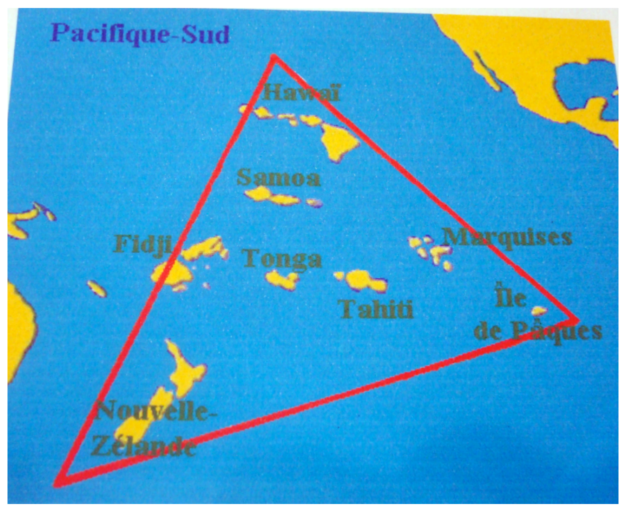
Sarah Fearnow

Suggested Activity Sequence:

10 m. Pass out the *Triangles Mind Map* Handout, p. 7. Have the group brainstorm what comes to mind when they think of triangles. Ask:

Where do you see triangles in real life?

Students' answers in my class included slices of pizza, ice-cream cones, cone-shaped hats, and the Bermuda Triangle. I did not discuss the difference between triangles and cones with them at this point, but rather accepted cones as examples of triangles.



30 m. Pass out images from the Internet of triangles in real life, one per person. There should be a variety of equilateral, isosceles, and scalene triangles among the images. Some examples of real-life triangles you might find include triangles in roofs, steeples, fences, bridge trusses, scaffolding, and playground equipment. Pass out copies of the *Triangles Research Activity* Handout, p. 8, rulers, and protractors.

Participants trace the triangle image onto the handout. Tell them to trace only one triangle if, as in the example of bridge trusses, there is more than one triangle in the image. Participants then measure the lengths of the sides and the degrees of the angles of the triangle they traced.

Keep in mind when trying this activity with students that many students have never used a protractor before and need guidance in lining up the hole in the protractor with the vertex of the angle they are measuring and the line on the bottom of the protractor with one of the lines in the angle. If the angle lines do not reach the protractor, the students can be advised to extend the lines to more easily read the measurement of the angles. Students also often get confused which of the numbers on the protractor to use as the correct measurement. When I asked my students whether the angle was acute or obtuse, they were usually able to identify which of the numbers to use, as we had studied these terms previously in the angles lesson.

This activity takes longer for students than for teachers because teachers are typically more comfortable using the measuring tools.

As participants finish the *Triangles Research Activity* Handout, they can add their data to the class chart, which should look something like the following:

Student Name	# of Equal Sides in Triangle Measured	# of Equal Angles in Triangle Measured
John	3	3
Maria	0	0
Barbara	2	2
Miguel	0	0
Danielle	3	3
Alma	3	3
David	2	2

20 m. Bring everyone together

Ask:

What patterns do you see in the data?

Participants should recognize that triangles always have the same number of equal angles as equal sides.

Among the teachers: Decide who is going to share the interpretations and issues that arose on the TIAN-talk listserve.

Then, focus on the classroom

- How is this lesson similar or different to how you have taught triangles in the past?
- What might be the benefits and challenges to teaching triangles in this way?
- Where would you go from here?

Handout 5A—Triangle Mind Map

Make a mind map of words or pictures that come to mind when you think of *triangles.* Where do we see triangles in real life?

Handout 5A—Triangle Research Activity

1. Trace the triangle from your image here.

2. Measure the lengths of the sides and the degrees of the angles and label them on your triangle.
3. How many of the sides are of equal length?

4. How many of the angles are of equal length?

5. Add your data to the class chart.

Geometric Thinking
Activity 5B. Larger than Life

(This activity is adapted from EMPower's *Over, Around, and Within: Geometry and Measurement,* Lesson 4—Giant-Size.)

Goal: Explore similarity of shapes by scaling up some everyday rectangular objects.

Time estimate:

40 minutes

Focus:

Geometric thinking: similarity and scale

Materials:

- ✓ Handout 5B: *Giant Size*, p. 11
- ✓ Graph paper
- ✓ Scissors
- ✓ Rulers
- ✓ Yardsticks
- ✓ Common rectangular objects of different shapes (such as a business card, a dollar bill, a greeting card, book, etc. Include at least one square object)—one per pair
- ✓ Chart paper
- ✓ Markers
- ✓ Calculators

Preparation:

On each table, place scissors, calculators, rulers, and yardsticks. Have several everyday rectangular objects to choose from, one per pair. On the back of each object, place a post-it note on which you have written a "factor" by which the object should be enlarged (such as 10x, 2x, 1.5x, etc.).

This activity is designed for teachers to explore together before doing something like it with their students. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE FROM YOUR TEACHER GROUP OR STUDENTS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

20 m. Explain that the group is going to be exploring the "big idea" of similarity. Ask:

Where does similarity show up in real-life?

Distribute Handout 5B: *Giant Size*, p. 11. Divide participants into pairs/small teams. For each team, provide a sheet of chart paper and markers. Give teams about 10–15 minutes to complete the activity.

20 m. After charts are displayed, asked participants to examine each poster. Then bring everyone back together and debrief the activity. Below are some prompting questions for each part of the activity.

ABOUT THE TASK

- *Does each enlargement appear similar to the original? Why/why not?*
- *Which object appears to be enlarged by the greatest factor? By how much?*
- *How would you compare the original perimeter and area to the new perimeter and area?*
- *What generalizations can you make?*
- *What does it mean to "draw something to scale"? How do the ideas of scale and similarity connect?*
- *How would this task play out with an assortment of triangular shapes? How would perimeter and areas compare?*

ABOUT STRATEGIES USED

- *What were some of the strategies teams used to address the task (visually, numerically, algebraically?)*
- *At which van Hiele level did you operate to address the task? How could you use another level? (This makes sense only after reading the introduction to this bundle or one of the articles on the van Hiele theory.)*

ABOUT TAKING THE TASK TO THE CLASSROOM

- *How might you extend this lesson in the GED classroom?*
- *In what types of GED items does a sense of similarity come to play?*

> The point of the discussion is that we have a "sense" when objects look similar, but similarity can be tested numerically, algebraically, by measuring, or by laying objects on top of one another. Each way deepens our understanding of the "big idea."

Handout 5B—Giant-Size

1. As a team, choose one of the objects.

2. Enlarge the object the indicated number of times. Try to think of more than one way to enlarge the object.

3. Compare the dimensions, perimeters, and areas of the two objects.

4. Hang up your poster, displaying both the original object and the enlarged reproduction.

Geometric Thinking
Activity 5C. Don't Fence Me In!

Goal: This activity explores the relationship between perimeter and area.

Time estimate:

60 minutes

Focus:

Geometric thinking: perimeter and area

Materials:

- ✓ Handout 5C: *Don't Fence Me In!*, p. 15
- ✓ Handout 5C: *How Does Your Garden Grow?,* pp. 17–19
- ✓ Graph paper (0.5 cm)
- ✓ Colored pencils
- ✓ Scissors
- ✓ Tape or glue sticks
- ✓ Cm/in rulers

Preparation:

Read Beck, Shari A, Vanessa E. Huse, and Brenda R. Reed. "How Does Your Mathematical Garden Grow? "in *Mathematics Teaching in the Middle School*, 13:2 (September 2007): 69 – 76. http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2007-09-68a&from=B. In this article, middle school students move from determining perimeter and area to discovering functional relationships with perimeters and areas.

This activity is designed for teachers to explore together before doing something similar with their students. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE FROM YOUR TEACHER GROUP OR STUDENTS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

10 m. Begin by asking participants to think of times they have worked in a garden. What was some of the math they used? Allow them to share out per table (or small group). Then bring the groups together and ask for some examples of math used in gardening. Post the examples on flip chart paper for others to copy as they might spark activities for the classroom.

Explain that gardening could involve many math concepts and that the garden activity they will work on today focuses on perimeter and area although we'll look at others as well.

20 m. 1. Distribute Handout 5C: *Don't Fence Me In!*, p. 15, to each participant. Explain that you have a 100-foot roll of 2-foot high fencing, perfect for keeping rabbits out of the garden, but short enough to step over. You want to use the entire roll of fencing in a section of your yard that you want to reserve for gardening.

Before beginning, ask participants to think about the area of the garden within the fence. Ask:

> *Do you think that the area within the fence will be the same no matter how you design your rectangular shape?*

Then ask them to create a couple of different rectangular shapes using the entire 100-foot roll. Graph paper is really helpful.

2. When everyone has had a chance to create at least two different shapes, ask for some examples and write the length and width on the left half of the board (or chart paper). Leave room on the right side to post the area of each rectangle later.

3. Ask participants to calculate the area of each of their rectangles, either by formula or by counting the number of square units. Now, draw another column on the right half of the board (or chart paper) and list the area of each of the rectangles.

Ask:

> *Was your prediction correct? Does the area within the 100-foot fencing remain the same no matter what the shape?*

4. Now ask:

> *Can you notice any relationship between the dimensions and size of the area?*

If people seem stuck, you might encourage them to organize the dimensions and areas of the rectangles by area, either in increasing or decreasing order. Be sure to give all participants time to discover the relationship on their own!

Discuss their findings. They should have found that, the closer the shape is to a square, the larger the area. If no one created a square with their 100-foot roll, ask them to predict what the area would be, then ask them to draw the shape and determine the area. Add the results to the list on the board.

5. As an Extension ask:

Do you think there is a way to increase the area even more, using the same 100-foot fencing?

Allow participants to grapple with the idea. There are many ways to think about this, and many shapes to play with—what about a triangular garden? Or a hexagonal one? Or a circle?

Compare the area of the circle with a 100-foot circumference with the square with a perimeter of 100 feet. Then ask what implications this might have on the houses we build. Compare an igloo, a tepee, and a ranch house.

20 m. Now distribute Handout 5C: *How Does Your Garden Grow?,* pp 17–19.

10 m. Bring participants back together and discuss the math that they explored through this activity. Begin by asking about some of the strategies they used to determine the area of their shapes. Ask for volunteers to share their strategies on the board and then ask them to compare the different ways area was calculated.

If there is time, ask them to create a GED-type problem based on what math they used in their garden design.

Handout 5C—Don't Fence Me In!

You have a 100-foot roll of 2-foot high fencing. This is perfect for keeping rabbits out of the garden, but it is short enough to step over. You want to use the entire roll of fencing in a section of your yard that you want to reserve for gardening.

1. Draw a rectangular shape in the grid paper that uses all 100 feet of fencing. Now draw 2-3 more rectangular shapes of different sizes, each of which uses all 100 feet of fencing. Be sure to label each side and check to make sure the total fencing is 100 feet.
2. Do you think the gardening space is the same for each of the shapes you drew?

3. Check out your prediction by figuring out the area of each shape. Put the total square units beside each of your shapes.
4. What did you discover?

Handout 5C—How Does Your Garden Grow?

Use the garden shapes to design a garden. Cut out and tape your shapes on the grid below. Create a scale based on what you would like your actual garden size to be. Don't forget to add pathways to walk between areas and rows.

Complete the questions on the next page.

Scale: ½″ =

1″ =

1.a. Based on your scale and garden shape, what is the total area, including pathways?

b. Show how you figured out the area.

c. Now think of another way to calculate the area.

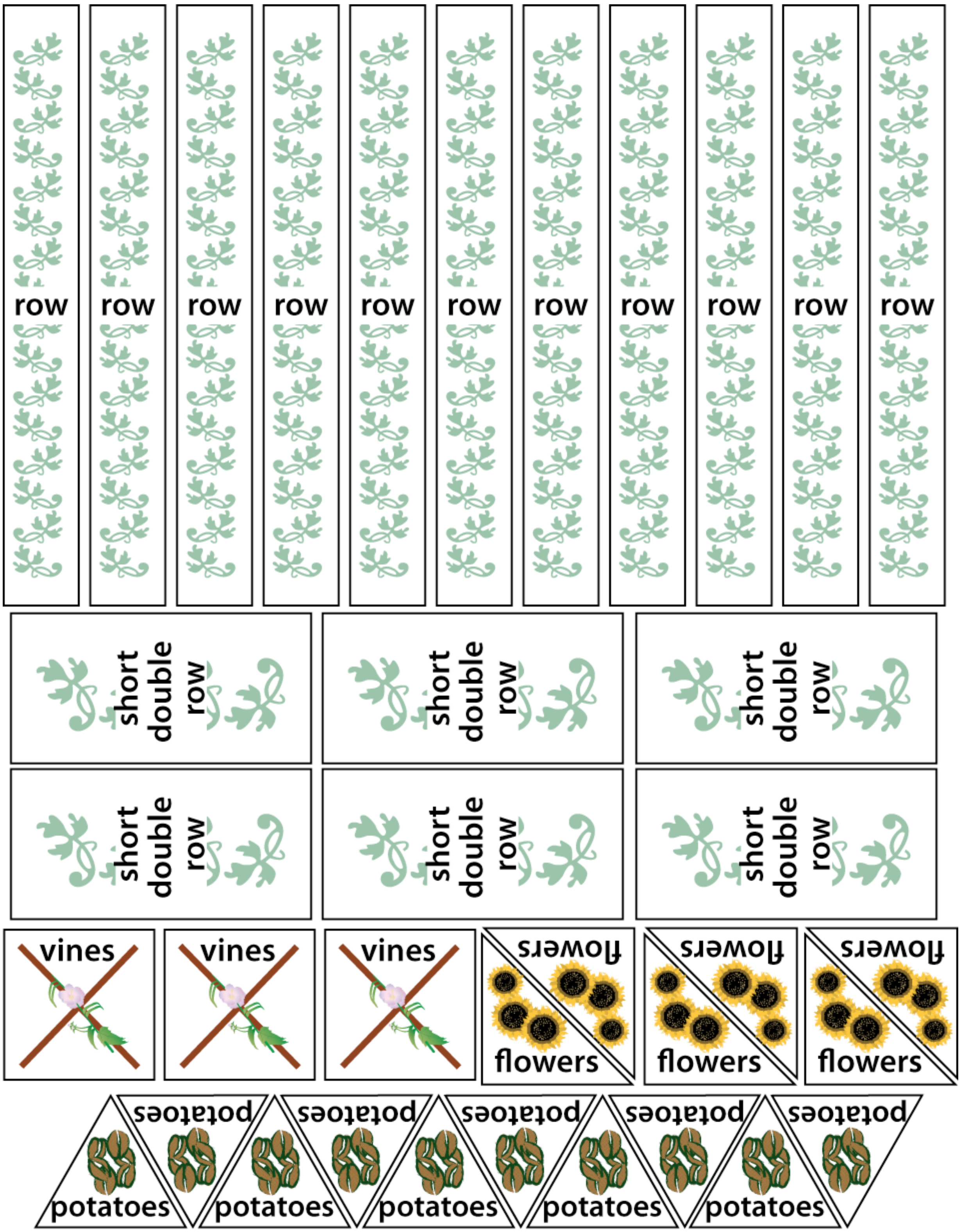
2. If you put a fence around the entire outside of the garden, how much fencing would you need (based on your scale)?

3. What if you decided to separate each garden area (according to the different shapes you used) with edging so that the design is clearly seen. How much edging would you need (based on your scale)?

4. Is the amount of edging more or less than the fencing? Explain your answer.

Challenge: Is there a way you could redesign your garden, using the same shapes, to save on fencing?

Garden Shapes



Geometric Thinking
Activity 5D. Geometry and Quilting

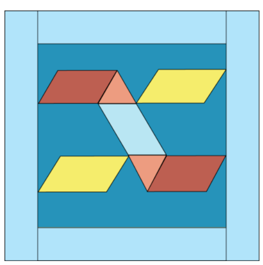
Goal: Teacher-participants share ways to use quilting as a context for geometry lessons, and experiment with one activity on symmetry.

Time estimate:

60–90 minutes

Focus:

Geometric thinking: symmetry



Materials:

- ✓ Article: *Shapes and Stitches: Quilting in an ABE Math Class*
- ✓ *Geometry and Quilting Lesson Template*, p. 24
- ✓ Handout 5D: *Geometry and Quilting: Symmetry*, p. 25
- ✓ Colored pencils
- ✓ Graph paper
- ✓ Pattern block shapes

Preparation:

Read the article, *Shapes and Stitches: Quilting in an ABE Math Class.* Distribute it to group members or have them download it from the TIAN website, http://adultnumeracy.terc.edu/pdfs/shapes_stitches.pdf. Ask them to read it ahead of time and to bring any ways they or colleagues have used quilting in the math class.

This activity is designed for teachers to explore together before doing something similar with their students. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE FROM YOUR TEACHER GROUP OR STUDENTS ON THE “TIAN Talk” DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

30-45 m. PART I: Sharing ideas about how to use quilting in math class

Prompt the discussion about the article *Shapes and Stitches: Quilting in an ABE Math Class* with questions such as:

What struck you about Linda Huntington's article?

What ideas did Linda's teacher-research give you about how to use quilting in your own class?

Have you quilted yourself or done quilting projects with your students?

What are math topics that would be good to explore with quilting—and how?

Capture their comments on chart paper, generating a list of activities and math concepts/skills.

If appropriate, have available the state's ABE math standards to see where such activities and concepts/ skills appear, or use the standards to help generate ideas.

For example:

Activity	Math concept/skill
Use pattern blocks to design a quilt block	✓ manipulate triangles, hexagons, trapezoids, parallelograms and squares and build confidence using the vocabulary and description of the shapes ✓ examine shapes for symmetry ✓ explore transformations: (reflection (flip). rotation (turn), translation (slide).

At this point, the group might decide to break up into small groups to develop lessons everyone might try out—at various van Hiele levels—and come back to share what happened at the next meeting. A Lesson Template is included here on p. 24.

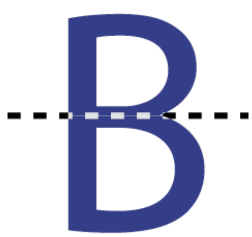
30-45 min PART II Geometry and Quilting Lesson: Symmetry

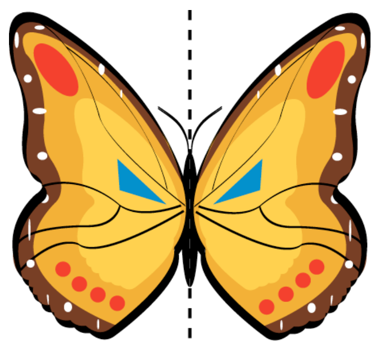
This quilting activity focuses on symmetry, but quilting could involve many other math concepts.

Step 1. This activity begins with exploring the idea of symmetry. Distribute Handout 5D*: Geometry and Quilting Symmetry* and ask people to describe symmetry using each different pattern block shape (square, triangle, hexagon, etc.). Have them draw lines of reflectional symmetry on the page of Pattern Block shapes then examine and discuss the rotational symmetry and the reflectional symmetry of each piece.

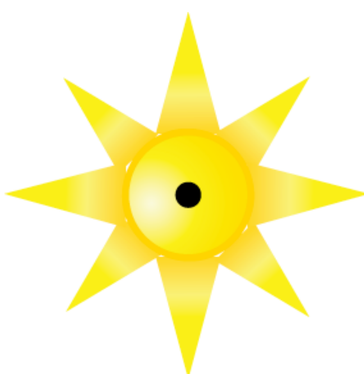
> ***Symmetry** is a geometric characteristic of both nature and art. You may already know the two basic kinds of symmetry, reflectional symmetry and rotational symmetry. A design has **reflectional symmetry** if you can fold it along a line of symmetry so that all the points on one side of the line exactly coincide (or match) all the points on the other side of the line… A design has **rotational symmetry** if it looks the same after you turn it around a point by less than a full circle. (Serra, M., p. 3).*

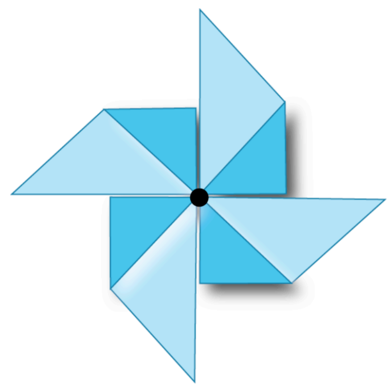
REFLECTIONAL SYMMETRY EXAMPLES





ROTATIONAL SYMMETRY EXAMPLES





Be sure participants are familiar with the symmetry terms, reflectional symmetry and rotational symmetry, by asking for more examples.

Step 2. Now examine and describe the symmetries in each of the three Pattern Block designs made by a quilter on pp. 26–28.

Step 3. Ask participants to take about 10 minutes to create a design of their own using just a few basic shapes. Then have them answer the questions.

Debrief the activity by asking about the geometry that they explored. Ask for connections with van Hiele's levels 0 (or 1, the lowest level) and 1 (or 2, the second lowest level) and discuss how, without exploration with shapes, students may not able to move from level to level.

Then ask participants to think of ways that this activity could be adapted to make it more challenging for students.

Geometry and Quilting Lesson Template

Activity Title: ________________________________

Goal: ____

Time estimate:

______ minutes

Focus:

geometric thinking—

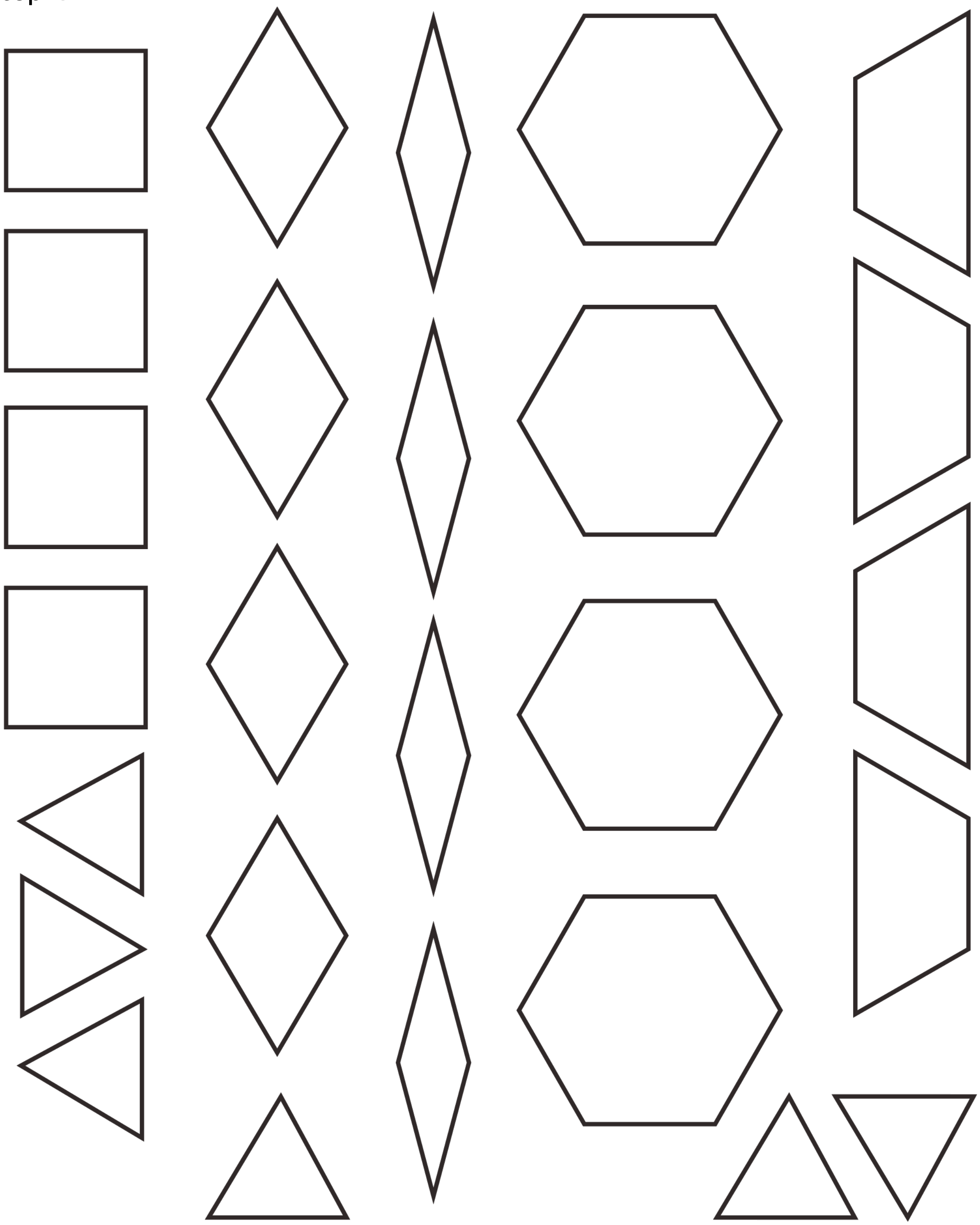
Materials:

Preparation:

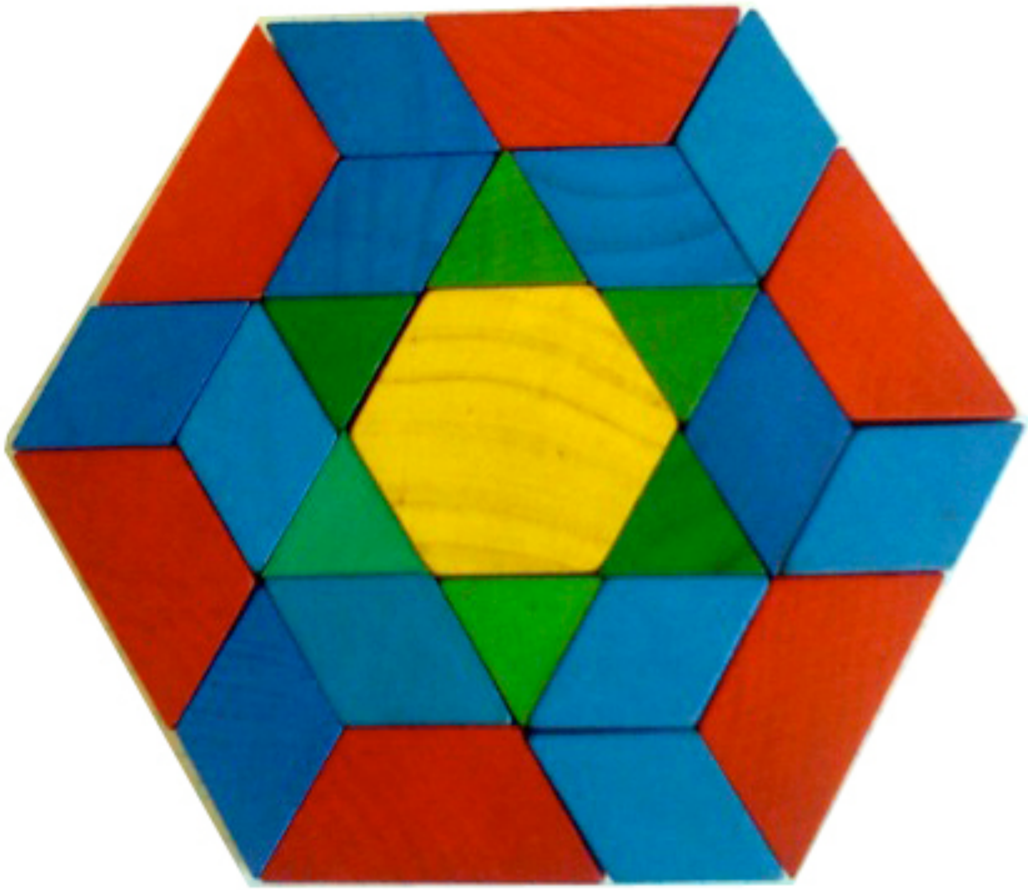
Suggested Activity Sequence:

Handout 5D—Geometry and Quilting Symmetry

Step 1

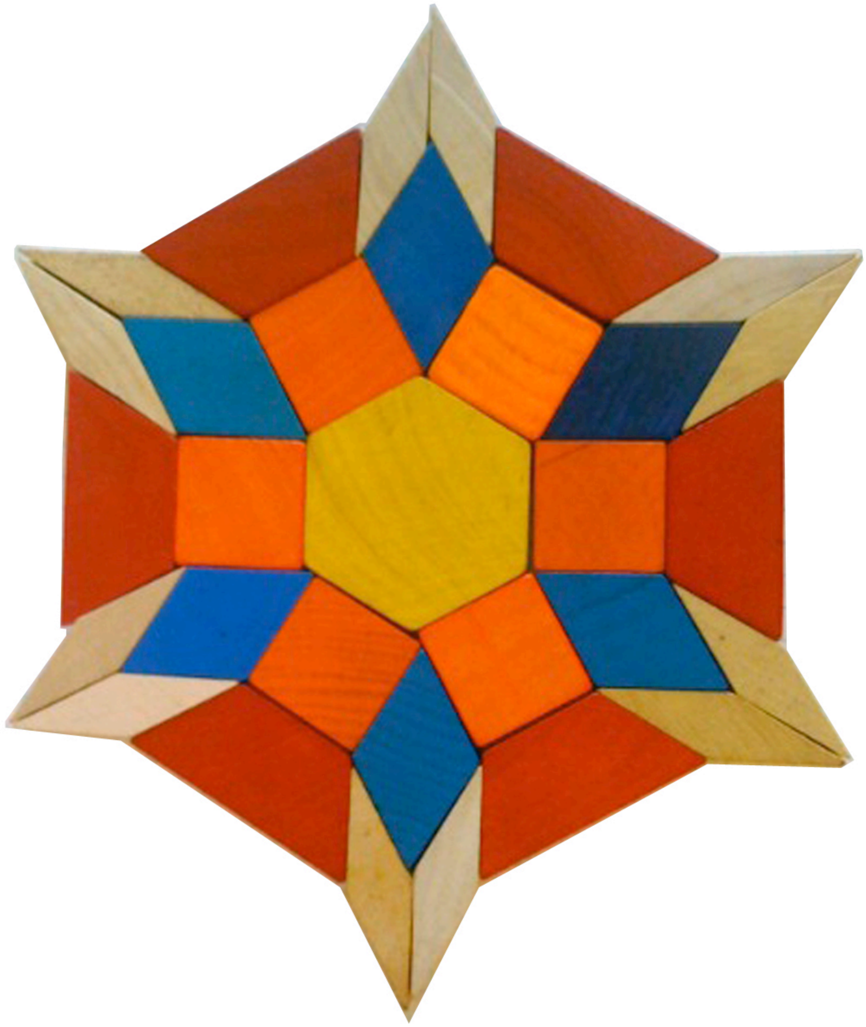


Step 2



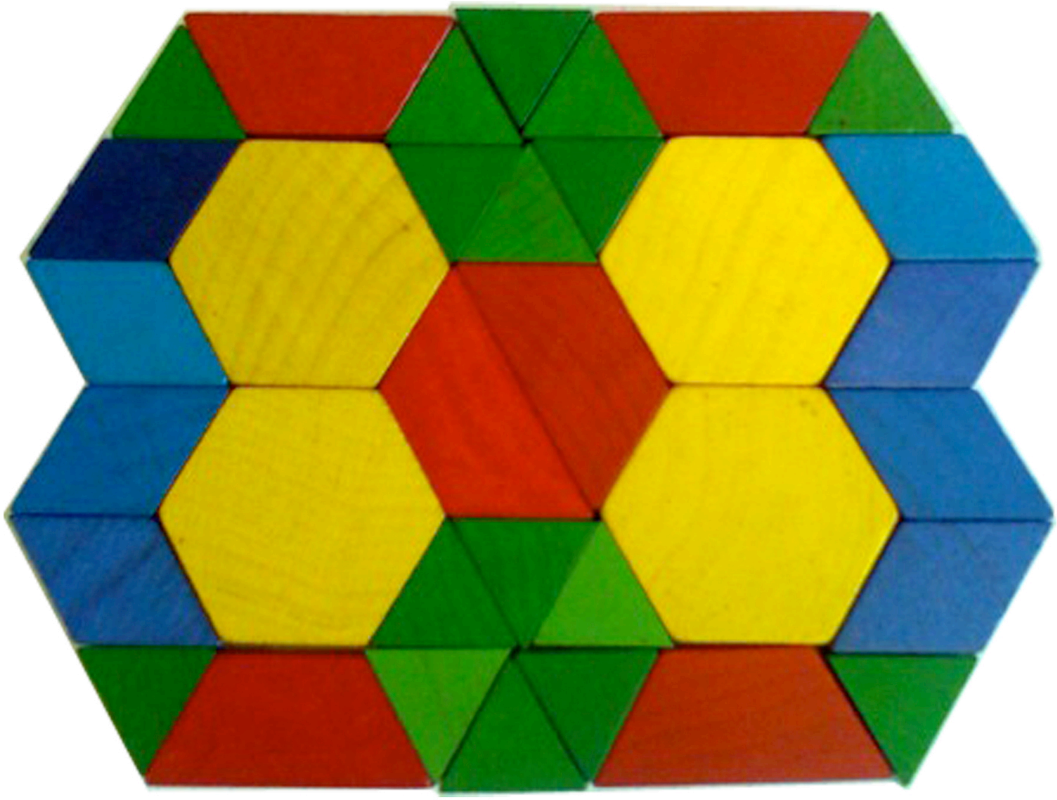
Can you see reflectional symmetry? Where?

Can you see rotational symmetry? Where?



Can you see reflectional symmetry? Where?

Can you see rotational symmetry? Where?

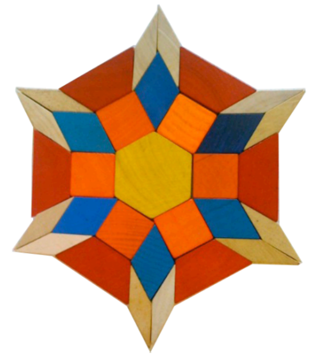


Can you see reflectional symmetry? Where?

Can you see rotational symmetry? Where?

Step 3

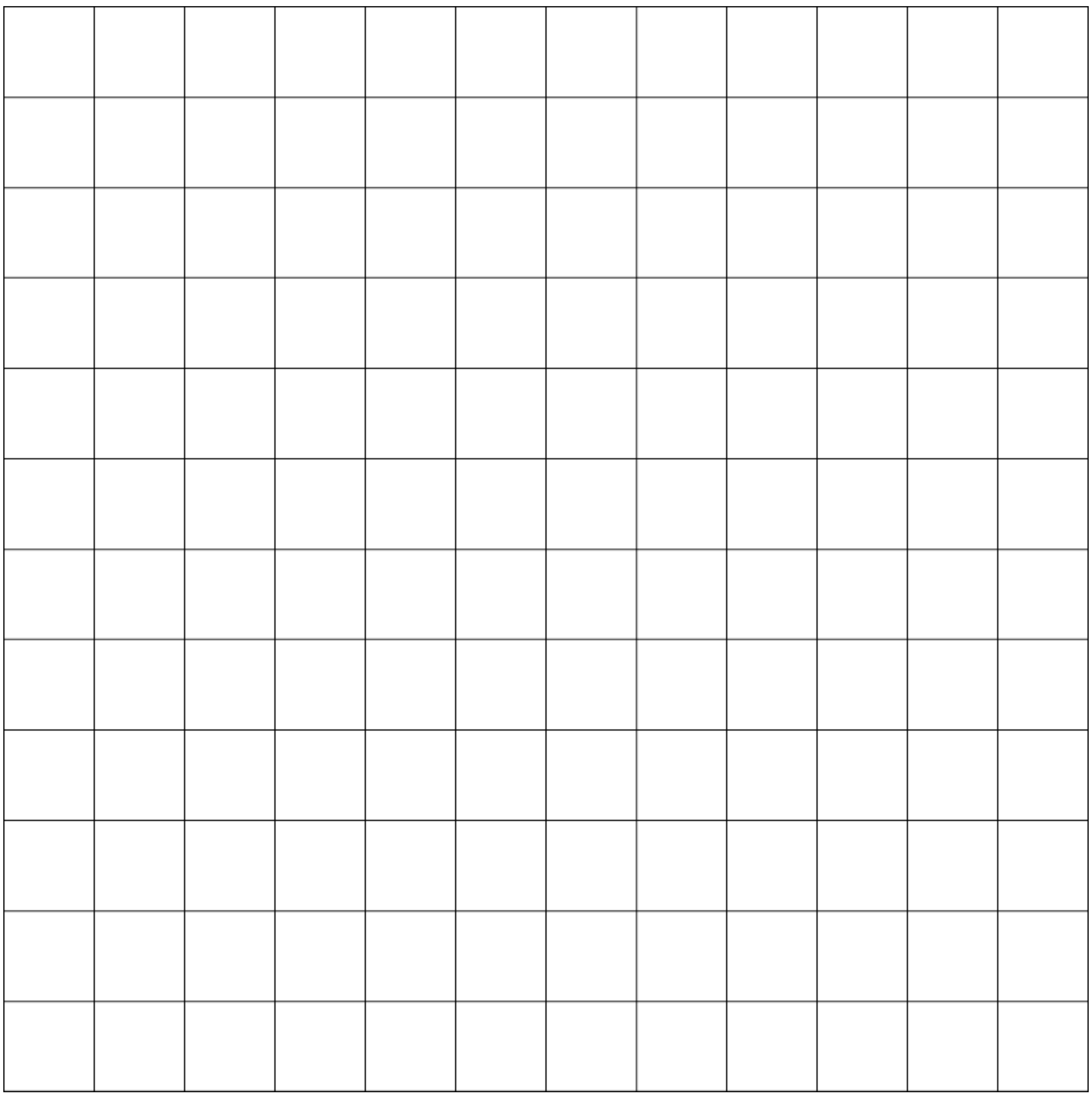
Create your own quilt square for a class quilt.



Design your quilt pattern in a 6” x 6” square on grid paper using any of the following shapes: small triangle, square, large triangle, rhombus, trapezoid, and hexagon.

Once the design is complete, label the square with the title of the design and the reasons for choosing the name or design. Choose any color for your quilt square.

Then answer the questions on the next page.



1. How many different shapes did you use? What are they?

2. Is your design symmetric? How so?

Geometric Thinking

Connecting to State Standards

The National Council of Teachers of Mathematics has emphasized that students should have opportunities to develop geometric thinking at all levels.

> Instructional programs from pre-kindergarten through grade 12 should enable all students to:
>
> - Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
> - Specify locations and describe spatial relationships using coordinate geometry and other representational systems
> - Apply transformations and use symmetry to analyze mathematical situations
> - Use visualization, spatial reasoning, and geometric modeling to solve problems
>
> *Principles and Standards for School Mathematics* (2002), NCTM p. XX.

Check out the geometry portion of your state's ABE Math Standards.

1. Is geometry included at all levels, as NCTM suggests?
2. What aspects of geometry are included?
3. How does the treatment of each aspect of geometry differ from level to level?
4. Do you think the geometric emphasis at each level is correct? Are there any revisions you would suggest to strengthen your state standard's treatment of geometry?
5. Check out another TIAN state's standards listed in the links below—do you notice any differences?

Below are links to each of the 6 states standards web pages.

Arizona

http://www.ade.az.gov/adult-ed/adult_ed_standards.asp

Kansas

http://adultnumeracy.terc.edu/pdfs/KS_state_standards.pdf

Louisiana

http://www.doa.louisiana.gov/osr/lac/28v129/28v129.doc.

Massachusetts

http://www.doe.mass.edu/acls/frameworks/

Ohio

http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&TopicRelationID=966&Content=21875

Rhode Island

http://www.brown.edu/lrri/standards.html,

Articles and References (For Teachers) About Geometric Thinking

Read and discuss some of these articles with fellow teachers. Use the attached Research Reading Response (p. 35) to help guide your discussions. Please share your thoughts with others by posting online to the TIAN Talk discussion list at tian-talk@cls.coe.utk.edu.

van Hiele:

Malloy, Carol. The van Hiele Framework. http://www.aug.edu/~lcrawford/Readings/Geom_Nav_6-8/articles/geo3arn.pdf. Provides an explanation of van Hiele's model.

"Van Hiele Levels of Geometric Reasoning" in IMAGES (Improving Measurement and Geometry in Elementary Schools. http://images.rbs.org/cognitive/van_hiele.shtml. Provides some concrete examples of various levels of the van Hiele model.

Woleck, Kristine Reed. "Tricky Triangles: A Tale of One, Two, Three Researchers" in *Teaching Children Mathematics*, September 2003: 40 -44. http://my.nctm.org/eresources/view_media.asp?article_id=6269. While the researchers focus on first grade, the article illustrates the sort of thinking at van Hiele's lowest level of geometry.

Triangles:

Maxwell, Sheryl A. "Measuring Tremendous Trees: Discovery in Action" in *Mathematics Teaching in the Middle School*, 12:3 (October 2006): 132 – 139. http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2006-10-132a&from=B. The "shadow" problem comes to life in this article about measuring the height of a tree using a handmade clinometer.

Reynolds, Marie J. "Letting the Cat out of the Bag....to Make Room for a Triangle" in *Mathematics Teacher*, 95:1 (January 2002): 6 – 7. http://my.nctm.org/eresources/view_media.asp?article_id=1841. An interesting activity to introduce the idea of congruence – having groups of students figure out the shape of a triangle in a bag.

Perimeter and Area:

Beck, Shari A, Vanessa E. Huse, and Brenda R. Reed. "How Does Your Mathematical Garden Grow? "in *Mathematics Teaching in the Middle School*, 13:2 (September 2007): 69 – 76. http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2007-09-68a&from=B. In this article, middle school students move from determining perimeter and area to discovering functional relationships with perimeters and areas.

Pagni, David L. "Finding Areas in Dot Paper" in *Mathematics Teaching in the Middle School*, 12:5 (December 2006/January 2007): 274 – 278. http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2007-01-274a&from=B. Students using dot paper to find the area of rectangles, triangles, parallelograms, and trapezoids readily see how the various formulas were derived. Sample worksheets are included.

Other:

Cipoletti, Beth and Nancy Wilson. "Turning Origami into the Language of Mathematics" in *Mathematics Teaching in the Middle School*, 10:1 (August 2004): 26 – 31. http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2004-08-26a&from=B. Explains how to effectively use origami in the classroom to teach geometric vocabulary.

Roberts, Sally. "Not All Manipulatives and Models Are Created Equal" in *Mathematics Teaching in the Middle School*, 13:1 (August 2007): 6 – 9. http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2007-08-6a&from=B. Something to think about as you use readily-available manipulatives in your class.

Tepper, Anita Benna. "A Journey through Geometry: Designing a City Park" in *Teaching Children Mathematics*, February 1999: 348 – 352. http://my.nctm.org/eresources/view_media.asp?article_id=1336. The description of an integrated curriculum project that would be applicable to students of all ages.

Morris, Barbara. "The Beauty of Geometry" in Mathematics Teaching in the Middle School, 9:7 (March 2004): 358 – 361. http://my.nctm.org/eresources/view_media.asp?article_id=1336. A hands-on project that allows students to be creative while learning the vocabulary of geometry.

Research Reading Response

Title/Author/Reference:

Main Ideas:

- What were the author's main ideas regarding adults/children *learning* mathematics?

- What were the author's main ideas regarding *teaching* mathematics to adults/children?

Applications:

- What are the implications for you as an ABE mathematics teacher?

Classroom Resources
To Strengthen Geometric Thinking

This is a starter list of classroom resources that focus on geometry using models, hands-on investigations, and real-life experiences. If you know of other resources, please share with others by posting online to the TIAN Talk Discussion list at tian-talk@cls.coe.utk.edu.

Published and Online Resources

NCTM Illuminations Lessons

Illuminations activities [http://illuminations.nctm.org/ActivitySearch.aspx] are packed with interactive lessons for grades K–12, many of which can be adapted for adult learners. There are several lessons that investigate the areas of different shapes (parallelograms, trapezoids, and triangles) as the base and height change. See http://illuminations.nctm.org/ActivityDetail.aspx?ID=108 for the activity on parallelograms. Cube nets help students visualize two dimensional shapes which can be turned into three-dimensional figures: http://illuminations.nctm.org/ActivityDetail.aspx?ID=84.

NOVA Online

NOVA has quite a few interesting activities that integrate science and math. For example, the activity "Build a Bridge" (http://www.pbs.org/wgbh/nova/bridge/) has students applying geometric principles to decide which bridge would be most appropriate for different landscape situations.

EMPower Books

Most of the lessons in ***EMPower's Over, Around, and Within: Geometry and Measurement*** provide opportunities to explore angles, shapes, perimeter, area, and volume. More information and a sample lesson are available at: http://www.keypress.com/x18180.xml.

Discovering Geometry: An Investigative Approach

by Michael Serra is a great resource for more indepth geometry lessons and explorations. See http://www.keypress.com/x5233.xml for information and sample lessons.

Patty Paper Geometry

By Michael Serra

Discovering Geometry author Michael Serra brings discovery through paper-folding activities to the classroom with his exciting book *Patty Paper Geometry*.

http://www.keypress.com/x5837.xml

NLVM

The Virtual Manipulates website at http://enlvm.usu.edu/ma/nav/bb_school.jsp?sid=emready&coid=geometry has quite a variety

of geometry activities. Activities range from geometric transformations to right angle geometry and trigonometry. See for yourself!
http://mathforum.org/sum95/suzanne/whattess.html

Pack It Up

The website at http://www.nsa.gov/teachers/ms/geom_packit_up.pdf includes a series of three lessons on geometry and measurement. Students first analyze different solid objects. They then learn how to find the surface area and volume of rectangular prisms. Lastly, they apply their knowledge in a real-life activity related to package design.

Free Federal Resources for Educational Excellence

This website offers a variety of free resources on a variety of math topic, including geometry. http://free.ed.gov/subjects.cfm?subject_id=184&res_feature_request=0. Once you're in the geometry section, take a peek around, but be sure to check out the geometry learning units for middle school level at http://www.nsa.gov/teachers/teach00017.cfm —many activities appropriate to adults.

Ideas

Ordered Pairs Game

This is similar to Battleship in which an individual tries to figure out the correct coordinate points for an object. Pair students. Give each student a sheet of graph paper and have them fold it in two. On both halves, students will draw a first quadrant. [Later this can be extended to all four quadrants if students are ready.]

On the top half, have each to draw a polygon (such as a triangle, rectangle, or other regular figure) without letting the partner see the drawing. Suggest that students not draw something too challenging until everyone is comfortable with the process.

Once a figure is drawn, each student tries to figure out what shape her partner drew. To do this, the students take turns asking whether a point for the shape is at a particular location on the grid. The students answer yes or no to each question. Each student plots the point that she calls out on the bottom half of her graph paper. If the point is "yes", she places a dot on her paper. If the answer is "no", then she places a small x on that coordinate point (in order to keep track of which points have been called out and which haven't)

Once a student thinks she has the shape figured out, she calls out the ordered pairs.

Pattern Blocks Similarity

Ask students to use Pattern Blocks to create a shape. Then ask them to create a similar shape using the same sizes of Pattern Blocks. [For example, in order to make a larger square, students will have to use four squares for the next larger size square.] Ask them to compare the perimeter and area of the two shapes. Encourage them to rotate the shapes so that they can begin to see that rotated shapes can still be similar.

Decorating the House

You might ask students to bring in pictures of their own homes and determine the perimeter using scale (you might want to assign the scale even though it won't be perfectly accurate). Then ask them to plan to trim the house in holiday lights. Students will need to decide how many strings of what length and how many lights per string in order to decorate their house.

Cartoon Project

Students need to bring in one square of a cartoon line, usually found in the comic section of a newspaper. They will use a ruler to draw vertical and horizontal lines (a grid) over the cartoon. The spacing between the lines should be about 1 cm. After the students have completed drawing the grid over the cartoon square, they will draw another grid over the entire sheet of white 8.5 x 11 in. paper. The spacing between the lines for this sheet should be about 1 inch. Now the students must draw free-hand, square by square on the white paper what is on the original cartoon. They will copy each square until the cartoon is completed and then color using the same colors that were in the original cartoon. They should cut off whatever white squares that were not used. Each student needs to mount the original cartoon along with the enlarged cartoon on a piece of construction paper. The students will probably not be able to finish the project in one class period. From http://eduref.org/cgi-bin/printlessons.cgi/Virtual/Lessons/Mathematics/Measurement/MEA0210.html

Attendance Turbulence

Throughout the TIAN Institutes and activities between sessions, group work was modeled and encouraged. Sometimes activities that participants, and then students engaged in, took longer than one class period. Everyone seems to agree that group work and longer activities are more effective when students consistently attend, but they present a challenge when student attendance is erratic.

In all the TIAN Institutes, we asked the question: what makes teaching in a “TIAN-like” way challenging? Often, we heard that “erratic attendance” or “open enrollment” or “drop-in labs” were challenges. While there was important discussion about the challenges and reasons for those, the most valuable discussion occurred when teacher-participants offered solutions to the attendance challenges. We have included those suggestions which we encourage you to share with participants once they have done their own brainstorming.

Attendance Turbulence Activity:

1. In small groups, pose the following questions:

   How do you deal with erratic attendance in your class?

   If you have a lab situation, how might you modify it in order to allow more group work and TIAN-like activities?

2. Ask each group to share their ideas.
3. Distribute the handout: *Suggested Solutions for Dealing with “Attendance Turbulence” in TIAN Classrooms*. Give participants time to read over the list generated by participants from the four TIAN field test states.
4. Discuss how their brainstormed list compares to that of the field test sites. Ask:

   Which strategies do you think would be most workable for you in your situation?

   If you asked your students, what solutions would they come up with?

Suggested Solutions for Dealing with “Attendance Turbulence” in TIAN Classrooms

(This list was generated by participants from the four TIAN field test states.)

- Have a student volunteer to briefly recap the previous lesson for those who were absent
- Have packets of previous hand-outs available for absent students
- Each lesson should be able to stand alone as an engaging activity (not bookwork)
- Explore moving students to other classrooms or tutors or other hours of the day based on geography, personal issues, etc.
- Think individually and plan strategies with individual students
- Save the lessons for the days that have best attendance
- Promote involvement in an activity that has national implications
- Build relationships with teachers, other students (bonding)
- Opportunity/incentive to show them success… accountability
- Self-contained lessons to one period
- Have a “suggested” daily schedule for students to follow so as to keep them attending all day rather than an hour
- Have a calendar/folder for work missed
- Rewards for attendance
- Post weekly/monthly schedule of small group activities
- Create reviews to act as a bridge to next lesson
- Reward system—brownies, certificates, coupons
- Charge for classes
- Recruit (mail) math challenged (passed all GED but math) students
- Require tuition/refunded upon completion or attainment of gains
- Schedule regular standard lessons
- Mandate lesson time and immediate assessment of lesson content
- Attendance contracts
- Positive reinforcement

- Upfront expectations
- Goal setting individually with follow-up
- Phone calls

Lab/Individualized

- Ask to be a "guest teacher" with a regular classroom/class to be able to teach EMPower lessons
- "Team teach" with a teacher who has a traditional class/classroom setting
- Ask students who are there if anyone wants to get together as a small group and "do a fun math activity" together (2-5 students) (right then and there)
- Advertise these activities—set up time and date and invite students

Introduction to the Facilitator's Guide

Each TIAN Bundle's third section (the Facilitator's Guide) is designed to give some practical suggestions about how to facilitate a teacher meeting using the resources in the other two bundle sections (Math Topic and Teaching/Learning Issue). There is a suggested Meeting Feedback Form for the group and a Teacher Meeting Notes form to submit to the tian-talk discussion list by sending an email to tian-talk@cls.coe.utk.edu. Please note these are only suggestions. The TIAN team is interested in hearing what groups decide is most important and helpful for them.

Suggestions for Using Bundle #5 in Teacher Meetings

As you plan to use a Bundle, print out a copy of the entire Bundle (about 40 pages). Read through it, deciding which sections to photocopy for the meeting and which to let group participants access themselves on the TIAN website at http://adultnumeracy.terc.eu/TIAN_teacher_resources.html

If your group has ONE two-hour meeting to spend on Bundle #5, set aside at least 2/3 of the time for the Math Topic and 1/3 of the time on the Teaching/Learning Issue or discussing one or more of the articles from the Articles and References for Teachers. So, a meeting might go something like:

1. **Introduce the Math Topic, Geometric Thinking, either by**
   a. summarizing the main points in the Introduction or
   b. emailing the introduction and Linda Huntington's article, *Shapes And Stitches: Quilting In An Abe Math Class,* ahead of time to the group members, and then briefly discuss the main points in the meeting.
2. **Do some math together.**

   There are four activities. Choose two activities to do such as 5A—*Triangles in Real Life* for a look at the properties of triangles and Activity 5C—*Don't Fence Me In!* to look for patterns and informal rules for perimeter and area.
3. **Consider the issue: "Attendance Turbulence" or discuss how effectively your state's standards address the broader definition of geometric thinking.**
4. **Get some feedback on the meeting and ask a volunteer to send an email to tian-talk to share good ideas that came up in the meeting.**

Also, ask everyone to bring back to the next meeting what they did with these activities in their classes.

If your group has TWO two-hour meetings (4 hrs) to use Bundle #5, you might spend the entire first meeting on the Math Topic, and the second meeting discussing how things played out in class, ending that second meeting with a discussion of the Teaching/Learning Issue.

In the first meeting, you might have time to do and reflect upon all four activities, and to begin to choose some articles to read before the next meeting. You might start the second meeting with everyone sharing their feedback based on the article(s) they read.

Continuum of Sociomathematical Norms

	Less likely to promote mathematical learning ⟵		⟶ More likely to promote mathematical learning
Sharing	Ideas and solutions are shared with minimal or no explanation	Thinking is described, often in procedural terms	Explanations consist of a mathematical argument
Solution Strategies	Emphasis is on one single solution or strategy	Multiple strategies and solutions are described	Emphasis is placed on the relationships among multiple solutions and/or strategies
Confusion & Error	Confusion and mistakes are avoided or ignored, or are corrected by the PD leader	Confusion and mistakes are acknowledged in hopes of causing disequilibrium and change in understanding	Confusion and errors are embraced as opportunities to compare ideas, re-conceptualize problems, explore contradictions in solutions, or pursue alternative strategies
Questioning	The PD leader asks questions aimed at maintaining social order or eliciting specific responses	Both the PD leader and teachers raise procedural and/or factual questions about the mathematics	Both the PD leader and teachers raise questions that push on understanding of mathematics/ mathematical reasoning
Community	Work is generally done individually or ideas are shared through PD leader explication	teachers collaborate to find solutions to problems	Mathematical argumentation forms the basis of a generative learning process where individuals take responsibility for their own and the group's progress

(Adapted from Yackel & Cobb)

The Importance of Promoting Teacher Mathematical Learning

ABE math teacher groups get together for two main reasons—to get some good math teaching ideas and resources for their classrooms and to expand their own math knowledge. The activities that you do together begin with teachers wrestling with the problems themselves. As they struggle, some things you do as facilitator will be more likely to promote mathematical learning than others. All facilitators should keep these five important ideas in mind:

1. TIAN teachers value sharing solutions among themselves and encourage sharing in the classroom. When asking people to share, encourage people to explain their thought processes.
2. In the TIAN institutes we were always interested in more than one strategy, and whether we could see the connections between the strategies.
3. Regard confusion and error as learning opportunities—don't avoid it.
4. Raise honest questions that push on the math. This means it is ok to not have the answer to the questions posed. All of us are learners—that includes the facilitator.
5. It's a community—everyone should take responsibility for the learning.

These ideas, so beautifully presented in the table on the next page, would be good for everyone in the group to have a copy of right from the first meeting.

Meeting Feedback Form

(for the group and the facilitator)

What was the most effective part of the meeting today, and why?

What would you change for the next time? Why?

What pressing issues/topics would be good to address?

Teacher Meeting Notes

(To share with other groups on the tian-talk discussion list at tian-talk@cls.coe.utk.edu)

Date/time of meeting:

Group Title and meeting location (City or town, State)

Facilitator(s)

Number of participants present

Describe what occurred at the meeting

Did you use any activities or discuss the issue from the TIAN Bundles? How effective were the activities or discussion of the issue?

Did your group use resources others than those in the TIAN Bundles? If so, please describe (or attach).