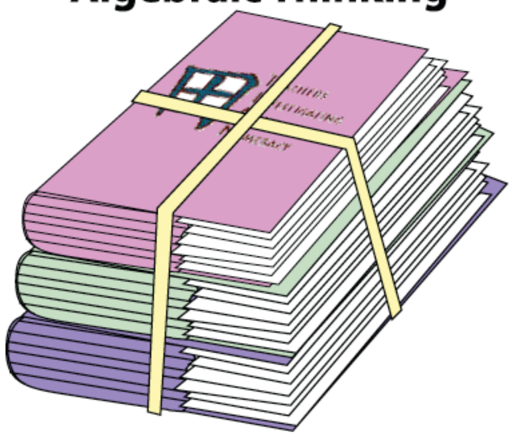
Teachers Investigating Adult Numeracy

Bundle #4
Algebraic Thinking



Contents

http://adultnumeracy.terc.edu/TIAN_teacher_resources.html

ALGEBRAIC THINKING
Introduction

"Critical to algebraic thinking is the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined functional rules"
(Driscoll, 1999, p.2).

Most adults remember "taking algebra" as learning how to manipulate symbols, or as Jim Kaput comically put it, "as studying the last three letters of the alphabet". But the definition of school algebra and pre-algebra has expanded considerably over past years, and what we see in good contemporary middle school and high school classrooms looks very different from what was taught even a few decades ago. While the how and why of the transformation of symbols (e.g., transforming $[x+5]\ [x+6]$ to $x^2 + 11x + 30$) is still very important, there is also a strong emphasis on understanding patterns, relations, and functions; on representing and analyzing mathematical situations using *multiple representations* (algebraic symbols, tables, graphs, and diagrams); and on creating and using *mathematical models* to represent and understand quantitative relationships.

We believe adult education teachers and their students who attended high schools where the emphasis was on symbol transformation will benefit from more opportunities to explore patterns, make generalizations, link the pattern to a function, and create multiple representations for any function. Bundle #4 provides adult ed teachers and their students with a bank of problems focused on three types of explorations.

Activity 4A: More Problems Like "Banquet Tables" has problems to explore patterns and make generalizations about physical objects. These problems also provide opportunities to link the multiple representations of the functional relations.

Activity 4B: More Problems Like "Phone Plans" includes problems that get at the algebra of "realistic" situations, providing opportunities to use a combination of tables, graphs, diagrams, and equations to understand and generalize about functional relations and to link the multiple representations.

Activity 4C: Using Algebra to Model Data consists of problems where students explore relationships between two variables—and see if they can create a mathematical model and line of best fit that describe the relationship fairly well. In several of these activities, students have to collect and organize their own set of data before graphing and analyzing it.

Each activity (4A, 4B, and 4C) contains several explorations for a group of teachers to select from and do themselves, and then to consider how to bring the activities to the classroom, and think about questions that might foster algebraic thinking.

There are many different ways to think about how to create a more expansive approach to algebra in the classroom. We have included the readings we have found very helpful in the *Articles and References*. Mark Driscoll's and Shelley Kriegler's works are particularly insightful.

Last but not least, please help us build this bank of problems—email tian-talk@cls.coe.utk.edu.

Have fun!

Algebraic Thinking
Activity 4A. More Problems Like "Banquet Tables"

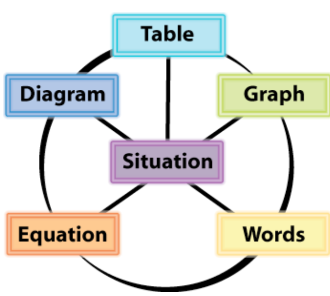
Goal: Teacher-participants explore some "classics" from the genre of problems which includes investigations such as "Banquet Tables". These problems ask people to explore patterns and make generalizations about physical objects. They also provide opportunities to link the multiple representations of the functional relations. Teachers consider the teaching implications of this type of problem, and add to their bank of algebraic problems.

Time estimate:

60–90 minutes

Focus:

Algebraic Thinking: Generalizing about Functional Relations & Linking Multiple Representations



Materials:

- ✓ *The Cake Problem* (p. 4)
- ✓ *The Painted Cubes Problem* (p 6)
- ✓ *Polygon Patterns* (p. 7)
- ✓ *Polyhedra Patterns* (p. 8)
- ✓ *Banquet Tables* (EMPower Seeking Patterns, Building Rules: Algebraic Thinking. Student Book, p. 21)
- ✓ *Toothpick Row Houses* (EMPower Seeking Patterns, Building Rules: Algebraic Thinking. Student Book, p. 26)
- ✓ *The Patio Project* (EMPower Seeking Patterns, Building Rules: Algebraic Thinking. Student Book, p. 147)

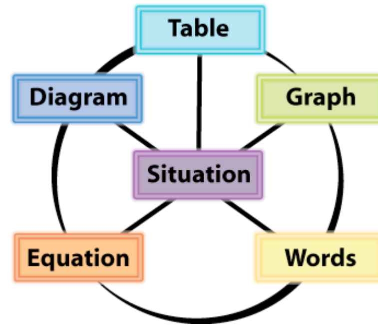
Preparation:

Try the activities yourself. Check each activity for materials needed, e.g., pattern blocks, toothpicks, marshmallows, etc.

This activity is designed for teachers to explore together before doing something similar with their students. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE FROM YOUR TEACHER GROUP OR STUDENTS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

10 m. 1. Introduce the Algebra Icon from EMPower. Ask teachers who participated in the TIAN institutes or who use the EMPower materials in their classes, to describe the “Banquet Tables” or the “Toothpick Row Houses” problem, vis a vis the Algebra Icon. For example, they may recall how they linked the equations or rule to the physical objects, showing how P = 2(T+1) connected to the people and the tables. They may also recall how important it was to make the connections between the physical model and the algebraic symbols.



Suggest that today we might:

a. Explore one or more of these types of problems;

b. Think about the math learning a person “gets” from that activity;

c. Consider the kind of questions that are good to ask students while they are doing these problems;

d. Decide whether the problem we did together can go directly to their classroom or whether it might need to be adapted.

30 m. 2. Working on the Problem Ourselves.
Distribute Handout 4A—Part 1, pp. 4–9. Ask the group to decide on one problem they would like to explore, then have people work in pairs on the problem. (Time varies with the problem chosen.) Bring the groups back together and have them share what they discovered as they explored.

30 m. 3. Thinking About Teaching.
Distribute Handout 4A—Part 2: Using Problems Like “The Banquet Tables” in Your Classroom, p. 10. Ask people to work in small groups to respond to the questions, then bring folks together to share their thoughts.

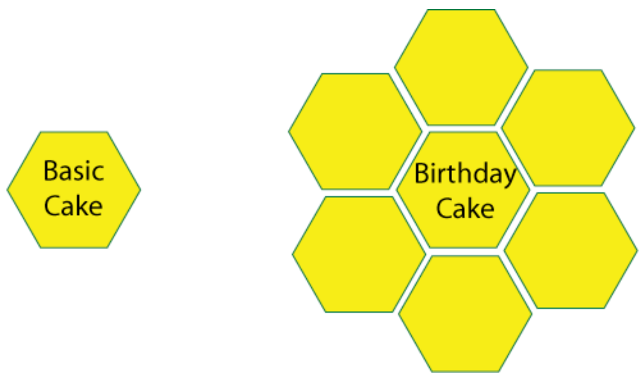
> To see how one teacher reported on her classroom experience with EMPower's "Banquet Tables" see http://adultnumeracy.terc.edu/TIAN_WS2_RI1.html For "Toothpick Row Houses", see http://adultnumeracy.terc.edu/TIAN_WS2_AZ1.html.

Handout 4A—Part 1

The Cake Problem

Materials
—Hexagonal pattern blocks
or
—Hexagonal grid paper, p. 5, also found at http://incompetech.com/graphpaper/hexagonal

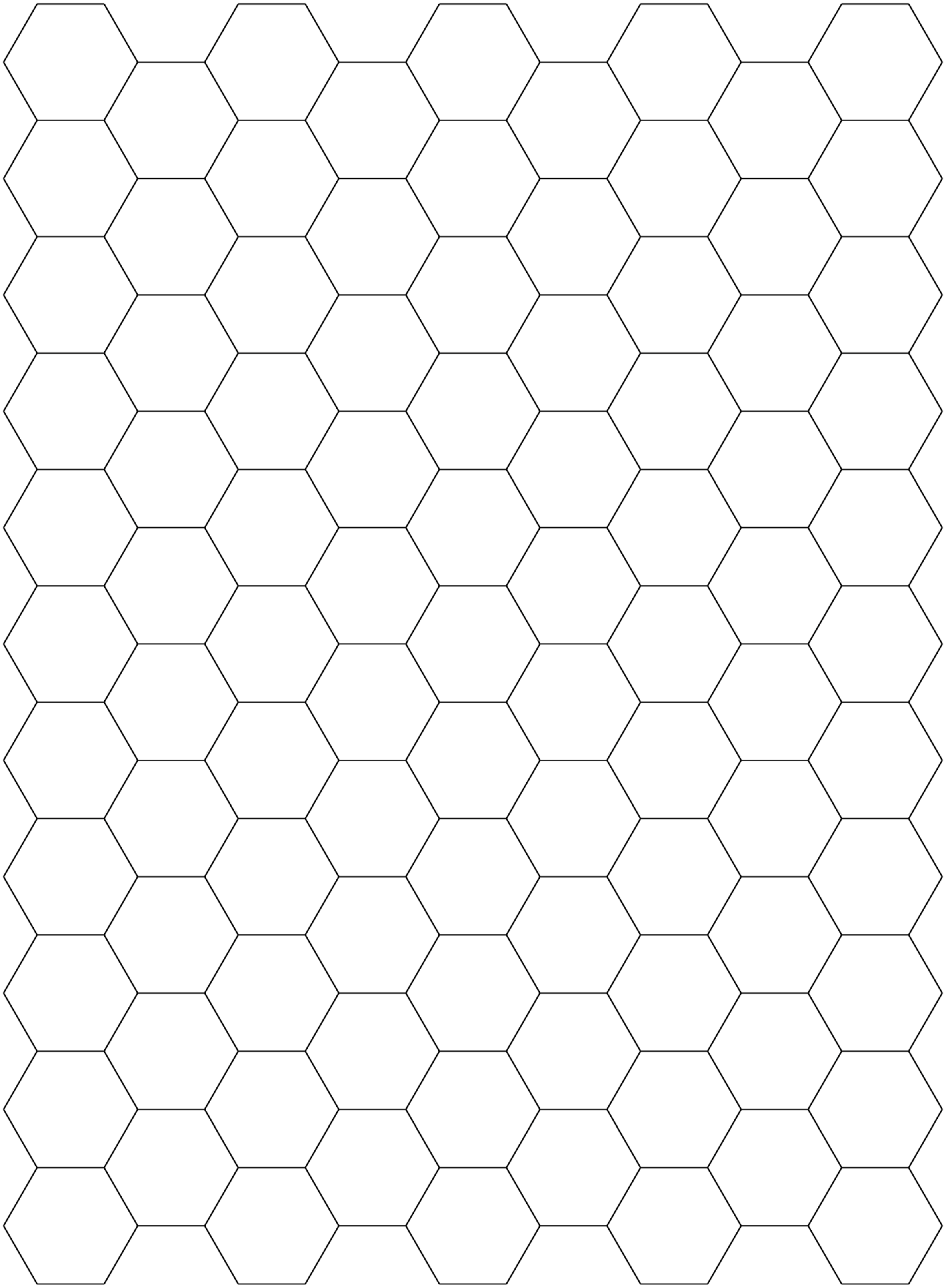
Distribute pattern blocks to students or copies of the hexagon grid on the next page. Explain that a bakery makes and sells hexagonal cakes. The "basic" cake, represented by the yellow hexagonal pattern block, sells for $6 and feeds five people. The cost of larger cakes is calculated on the basis of this "basic" cake. For example, a birthday cake is made by surrounding the basic cake with a ring of six additional cakes. Ask students to construct the birthday cake and to determine what a birthday cake would cost and how many people it would feed.



Repeat this process for a graduation cake that is made by adding a ring to the birthday cake, and for a wedding cake that is made by adding a ring to the graduation cake. Determine the cost of these two new cakes and how many people each would feed.

Ask students to complete a table of their data and to identify the relationship between the number of rings and the cost of each cake, and between the number of rings and the number of people each cake can feed. Next, ask students to find formulas that describe these relationships and to use these formulas to find the cost and number of people who can be fed if one created a record-breaking cake with 100 rings! Finally, students should be asked to create graphs of these relationships and to explain why one graph is straight and the other one curved.

[Adapted from A Guide to K-12 Program Development in Mathematics, Connecticut State Department of Education.]



Handout 4A—Part 1

The Painted Cube Problem

Imagine a large cube made up from 27 smaller red cubes. Dip the large cube into the yellow paint and complete the first row of the table. For an animation of this activity, visit http://nrich.maths.org/public/viewer.php?obj_id=2322

Materials
—Centimeter cubes or Unifix cubes

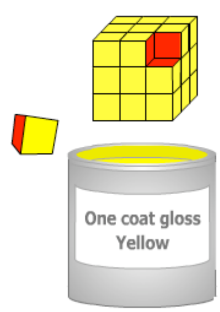
Outcome of dip

Size of large cube	No. of small cubes with 6 red faces	No. of small cubes with 5 red faces	No. of small cubes with 4 red faces	No. of small cubes with 3 red faces	Total No. of small cubes
3 x 3 x 3	?	?	?	?	27
4 x 4 x 4					
5 x 5 x 5					
6 x 6 x 6					
10 x 10 x 10					
23 x 23 x 23					

Imagine larger cubes being dropped into the paint and try to predict how this table would be filled in.

Can you see any patterns in the table?

Can you generalize these patterns?



How are they related to what you see?

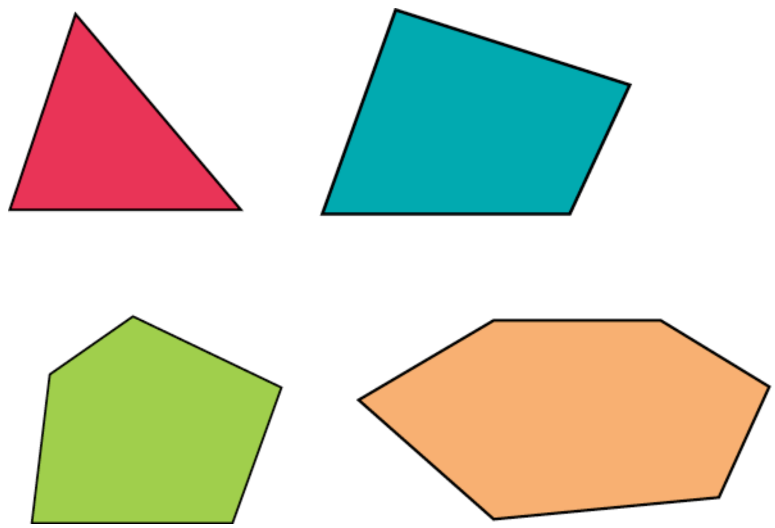
How would the table of results look for an *n* by *n* by *n* cube?

Handout 4A—Part 1

Polygon Patterns

The sum of the measures of the interior angles of a triangle is 180°.

1. Divide each of the polygons below into triangles to determine the sum of the interior angles.



2. Record your answers in the table below.

Sides	3	4	5	6	7	10	25
Sum	180						

3. What would the sum of the interior angles of a polygon with 102 sides be?

4. Determine a formula for the sum of the interior angles of a polygon with “n” sides.

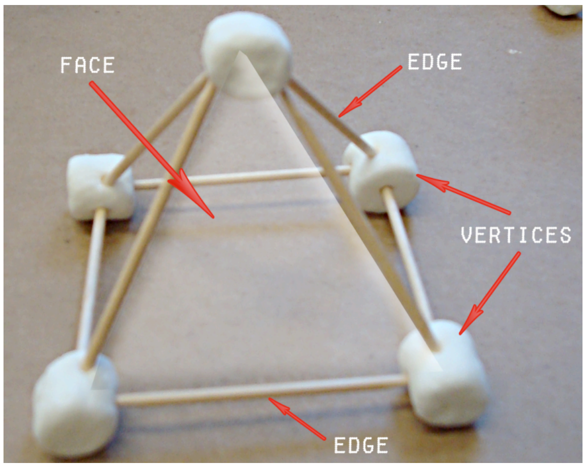
Handout 4A—Part 1

Polyhedra Patterns

Materials
—Toothpicks and miniature marshmallows
or
—Flexible straws and tape

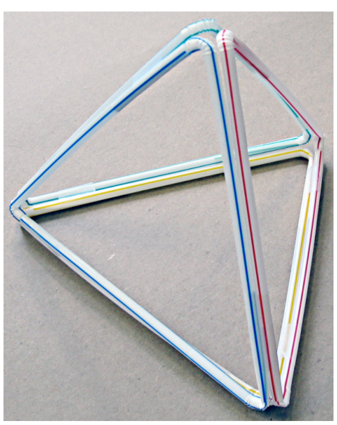
There is a relationship among the number of vertices, edges, and faces of a polyhedron. Can you detect it?

Definition: A *polyhedron* is a solid formed by polygons that enclose a single region of space.

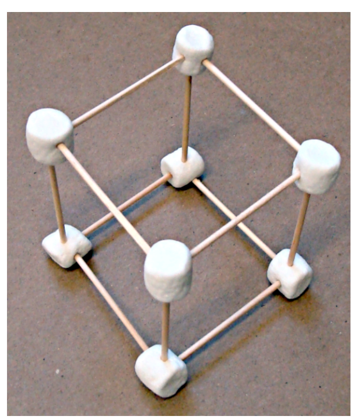


1. First, get up close and personal with some polyhedrons (or polyhedra). Build polyhedrons, using either toothpicks and miniature marshmallows or flexible straws. Build some of these:

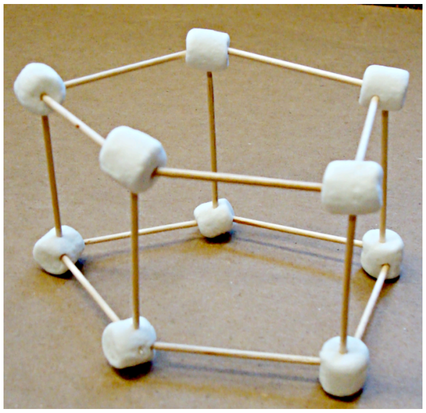
A pyramid (tetrahedron)



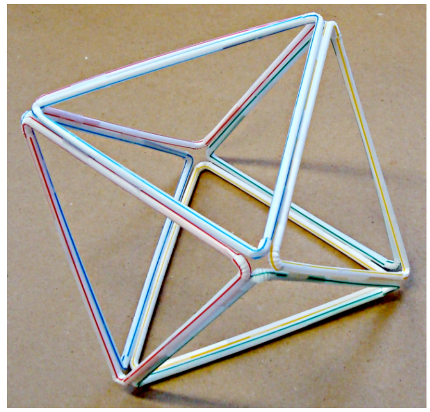
A cube (hexahedron)



A pentagonal prism



An octahedron



2. Now look for a relationship among the vertices, faces, and edges. Fill in the table.

Polyhedron	**Vertices (V)**	**Faces (F)**	**Edges (E)**
Pyramid	5	5	8
Cube			
Octahedron			
Decahedron			
Dodecahedron			

(As you examine the table or a pattern, fiddle around with the numbers. Try adding, subtracting multiplying or dividing V, F, and E to discover Euler’s Formula for Polyhedra.)

The formula is: ______________________________

What other patterns do you notice?

Extension: Go to this website for a great virtual view of the 5 regular polyhedrons rotating in space. http://mathforum.org/alejandre/applet.polyhedra.html

Handout 4A—Part 2

Using Problems Like "The Banquet Tables" in Your Classroom

1. Think about the problem you just worked on. What math learning do people get from doing this kind of problem?
2. Teachers who ask good questions while students work on these kinds of problems help students get the most "mathematical juice" out of the problems. The following are a list of questions suggested in the book *Fostering Algebraic Thinking* by Mark Driscoll. Which of these questions do you think are worthwhile for your students?

Questions for Building Rules to Represent Functions
Is there a rule or relationship here?
How does the rule work, and how is it helpful?
Why does the rule work the way it does?
How are things changing?
Is there information here that lets me predict what's going to happen?
Does my rule work for all cases?
What steps am I doing over and over?
Can I write down a mechanical rule that will do this job once and for all?
How can I describe the steps without using specific inputs?
When I do the same thing with different numbers, what still holds true? What changes?
Now that I have an equation, how do the numbers (parameters) in the equation relate to the problem context?

Can you think of other questions that would be useful?

3. Would you consider using the problem "as is" in your classroom? Why or why not? What adaptations, if any, would you make?

Check out how "Banquet Tables" worked in one teacher's classroom at http://adultnumeracy.terc.edu/TIAN_WS2_RI1.html

Algebraic Thinking
Activity 4B. More Problems Like "Phone Plans"

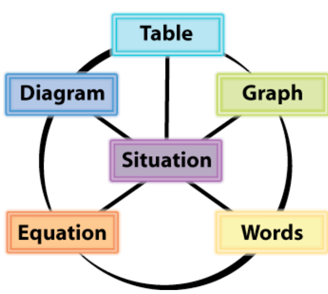
Goal: Teacher-participants explore some "classics" from the genre of problems that includes investigations such as "Phone Plans". These problems ask people to explore the algebra of "realistic" situations, providing opportunities to use tables, graphs, diagrams, equations to understand and generalize about functional relations and to link the multiple representations. Teachers consider the teaching implications of this type of problem, and add to their bank of algebraic problems.

Time estimate:

60–90 minutes

Focus:

Algebraic Thinking: Generalizing about Functional Relations & Linking Multiple Representations



Materials:

- ✓ *The Amusement Park* (p. 13)
- ✓ *The Walk-a-thon* (p. 14)
- ✓ *Let's Go Home* (p. 15)
- ✓ *The Big Trip* (p. 17)
- ✓ *Job Offers* (EMPower Seeking Patterns, Building Rules: Algebraic Thinking. Student Book, p. 101)
- ✓ *Phone Plans* (EMPower Seeking Patterns, Building Rules: Algebraic Thinking. Student Book, p. 113)
- ✓ *Rising Gas Prices* (EMPower Seeking Patterns, Building Rules: Algebraic Thinking. Student Book, p. 135)
- ✓ "Walking through Space" article

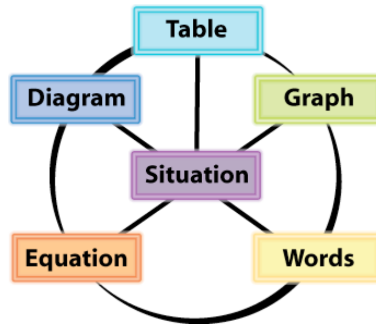
Preparation:

Try the activities yourself. Check each activity for a list of materials. Read the NCTM article, "Walking through Space."

This activity is designed for teachers to explore together before doing something similar with their students. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE FROM YOUR TEACHER GROUP OR STUDENTS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

10 m. 1. Refer the Algebra Icon from EMPower.
Ask teachers who participated in the TIAN institutes or who use the EMPower materials in their classes, to describe the "Phone Plans" or the "Job Offers" problem, vis a vis the Algebra Icon. For example, they may recall how they linked the features of the equations to the graph or the numbers in the in/out table.



Suggest that today we might:

a. Explore one or more of these types of problems;

b. Think about the math learning a person "gets" from that activity;

c. Consider the kind of questions that are good to ask students while they are doing these problems;

d. Decide whether the problem we did together can go directly to our classrooms or whether it might need to be adapted.

30 m. 2. Working on the Problem Ourselves.
Distribute Handout 4A—Part 1, pp. 13–19. Ask the group to decide on one problem they would like to explore, then have people work in pairs on the problem. (Time varies with the problem chosen.) Bring the groups back together and have them share what they discovered as they explored.

30 m. 3. Thinking About Teaching.
Distribute Handout 4A—Part 2: Using Problems Like "Phone Plans" in Your Classroom, pp. 20 Have people work in small groups to respond to the questions, then bring folks together to share their thoughts.

To see how one teacher reported on her classroom experience with EMPower's "Job Offers" see http://adultnumeracy.terc.edu/TIAN_WS2_KS2.html

Handout 4B—Part 1

Materials
—Graphing calculator
—Graph paper

The Amusement Park

Present the following situation to students:

> After exams, Juanita and her friends went to the amusement park. They found that each could buy an admission ticket for $5 and then pay 25 cents per ride. Their other option was to buy an admission ticket for $2 and then pay 75 cents for each ride.

- Select one of the following methods:
    a. Use a graphing calculator to plot each option. (Our note: This is optional. Adult ed teachers—are you aware that graphing calculators are used in many high school classes? Do you know how to use one?)
    b. Sketch a graph of each option, using the horizontal axis to represent the number of rides.
    c. Make a table with the number of rides in one column, and the cost of each option in successive columns.
    d. Write a system of equations and solve to find the number of rides that would cost the same under either option.
- Write a paragraph telling which would be the better deal and why.
- What factors should be considered in making a decision?

[Source: A Guide to K-12 Program Development in Mathematics, Connecticut State Department of Education.]

Handout 4A—Part 1

Walk-a-thon

Materials
—Graph paper

In her article, "Walking Through Space: A New Approach or Teaching Functions", by Mindy Kalchman found in the NCTM journal, Mathematics Teaching in the Middle School, a walk-a-thon is used to develop, compare, and contrast the patterns and properties of linear and non-linear functions.

The following problem was developed based on the examples in the article.

Antoine's class has decided to take part in a Walk-a-thon this spring to benefit the local animal shelter. Each member of the class has to find sponsors to pledge a certain amount of money for each kilometer walked in the Walk-a-thon.

a. The first sponsor pledged one dollar for every kilometer he walks in the 10 kilometer Walk-a-thon.

b. Another sponsor pledged $5 plus 50 cents a mile. How would you represent the $5 on a graph and in an equation?

c. A third sponsor established this rule: Multiply the number of kilometers you walk by itself. For example, when you have walked 3 kilometers, multiply 3 times 3 and you'll earn $9.

The class want to know how much would Antoine earn in each case after walking 1 kilometer; 2 kilometers; 3 kilometers; and 5 kilometers.

1. Make a table of values.
2. Make a graph to show Antoine's earnings in each case.
3. Predict what the graph will look like when all 10 kilometers are walked.
4. Represent each graph as an equation.

Work in pairs to come up with your own sponsorship scenarios and create the tables, graphs, and equations to model how they work.

Handout 4A—Part 1

Let's Go Home

Materials
—Graph paper

Juan and Lori just bought a house that needs to be remodeled. They are going to paint and carpet some of the rooms. Since they are on a limited budget, Juan began checking the advertisements.

Store	Paint Price
Home Hardy	$10 per gallon
Mix and Fix	Buy 3 gallons for $13 each, get one free
Sanders	Buy 1 gallon for $18; all additional gallons, ½ price

1. Complete the tables below so Juan can compare prices for 1 through 10 gallons of paint.

Home Hardy			Mix and Fix			Sanders	
In (# of gallons)	Out (price)		In (# of gallons)	Out (price)		In (# of gallons)	Out (price)
1			1			1	
2			2			2	
3			3			3	
4			4			4	
5			5			5	
6			6			6	
7			7			7	
8			8			8	
9			9			9	
10			10			10	

2. Where should Juan purchase the following amounts of paint?

   3 gallons? ______________________________________

   4 gallons? ______________________________________

   10 gallons? ______________________________________

3. Using three different colors, graph the paint prices.

Handout 4A—Part 1

The Big Trip

The Green family is planning a one-week vacation in Florida and needs to rent a car while there. They researched and found the following options. The Greens don't know exactly how far they will drive but estimate that it will be between 500 and 1000 miles. They must decide which plan to choose.

	*Weekly Rental	**Daily Rental
Axis Rental	$329 per week, unlimited mileage.	$50 per day, unlimited mileage.
Hersh Rental	$219 per week, plus 12 cents per mile.	$40 per day plus 3 cents per mile.
	**Partial week charged at a full week's price*	***Partial days charged at a full day's price.*

Make a Table

Comparison of Total Rental Car Costs per Week Based on Mileage Driven

Total Miles Driven	**500**	**600**	**700**	**800**	**900**	**1000**
Cost Weekly Axis Rental						
Cost Weekly Hersh Rental						
Cost Daily Axis Rental						
Cost Daily Hersh Rental						

Make a Graph

Using four colors, create a graph to compare costs.

Analyze the Graph

Do all of the points of each graph lie on a straight line?
Which of the lines is steepest? What is the slope of that line?
Which of the functions grows at the fastest rate?
Which of the functions grows at the slowest rate? What is its slope?
If you extend the lines through the y-axis, would any of the lines go through the origin? Explain why or why not.
What is significant about the points where the graphs intersect?

Write the Functions

Write the Total Week's Rental Car Cost as a function of the Number of Miles Driven for each of the options.

Total	**Function**
Cost Weekly Axis Rental	
Cost Weekly Hersh Rental	
Cost Daily Axis Rental	
Cost Daily Hersh Rental	

Write a Paragraph

Based on the best economics, explain under what conditions the Green Family should choose each option.

- Extension 1
  The Greens know that they will drive approximately 900 miles, but they don't know exactly how long they will be in Florida, only that it will be sometime between 3-8 days. Given these new circumstances, explain under what conditions they should choose each option.
- Extension 2
  The Greens don't know exactly how long they will be in Florida, only that it will be sometime between 3-8 days. They also don't know exactly how far they will drive, only somewhere between 500-1000 miles. Given these new circumstances, explain under what conditions they should choose each option.

Handout 4B—Part 2

Using Problems Like "Phone Plans" in Your Classroom

1. Think about the problem you just worked on. What math learning do people get from doing this kind of problem?
2. Teachers who ask good questions while students work on these kinds of problems help students get the most "mathematical juice" out of the problems. The following are a list of questions suggested in the book *Fostering Algebraic Thinking* by Mark Dricoll, p. XXX. Which of these questions do you think are worthwhile for your students?

Questions for Building Rules to Represent Functions
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Why does the rule work the way it does?
How are things changing?
Is there information here that lets me predict what's going to happen?
Does my rule work for all cases?
What steps am I doing over and over?
Can I write down a mechanical rule that will do this job once and for all?
How can I describe the steps without using specific inputs?
When I do the same thing with different numbers, what still holds true? What changes?
Now that I have an equation, how do the numbers (parameters) in the equation relate to the problem context?

Can you think of other questions that would be useful?

3. Would you consider using the problem "as is" in your classroom? Why or why not? What adaptations, if any, would you make?

Check out how "Job Offers" worked in one teacher's classroom at http://adultnumeracy.terc.edu/TIAN_WS2_KS2.html

Algebraic Thinking

Activity 4C. Using Algebra to Model the Data

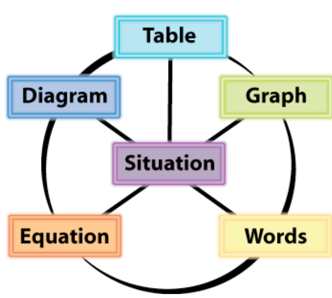
Goal: Teacher-participants explore some data sets to determine whether there is a linear relationship between two variables, and, when there is, to develop a *mathematical model* in the form of $y = mx + b$. Some of the problems provide small sets of data (Oil Changes and Engine Repairs, Weights and Drug Doses), while others require you to obtain the data, either by measuring (Circle Patterns, Tying Knots, Beam Strength) or by doing some research (Housing). Teachers consider the teaching implications of this type of problem, and add to their bank of algebraic problems.

Time estimate:

60–90 minutes

Focus:

Algebraic Thinking:
→Generalizing about Functional Relations
→Linking Multiple Representations



Materials

- ✓ *Oil Changes and Engine Repairs* (p. 24)
- ✓ *Weights and Drug Doses* (p. 25)
- ✓ *A Functional Housing Market* (found at http://www.spa3.k12.sc.us/house.html)
- ✓ *Circle Patterns* (from EMPower Seeking Patterns, Building Rules: Algebraic Thinking. Student Book, p. 77)
- ✓ *Tying Knots* (from Discovering Algebra, Key Curriculum Press, p. 204, http://www.keypress.com/x5265.xml Click on Chapter 3 link to get PDF of lesson)
- ✓ *Beam Strength* (from Discovering Algebra, Key Curriculum Press, p. 226)

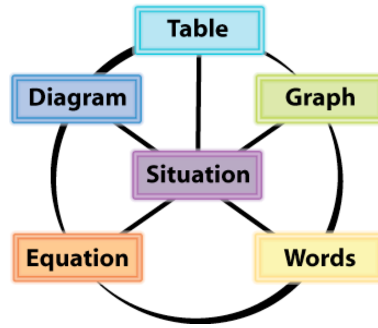
Preparation:

Try the activities yourself. Check each activity for materials needed.

This activity is designed for teachers to explore together before doing something similar with their students.
PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE FROM YOUR TEACHER GROUP OR STUDENTS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

15 m. 1. Introduce the Algebra Icon from EMPower. Ask teachers who participated in the TIAN institutes or who use the EMPower materials in their classes, to describe the "Circle Patterns" or "Tying Knots" problems, or the collection of personal data when we compared the time it took to travel to the institute and the distance traveled, vis a vis the Algebra Icon. They may recall how they detected a pattern in the tabulated data that was collected (e.g., the circumference is always about three times the diameter. They may also recall how important it was to make the connections between the physical model and the algebraic symbols and the graph.



Spend some time discussing the definition of "mathematical model." You might want to think about the ones we found when we did a Google search, on http://www.answers.com/topic/mathematical-model?cat=technology.

An accounting dictionary defined a mathematical model this way:
Mathematical representation of reality that attempts to explain the behavior of some aspect of it. The mathematical model serves the following purposes: (1) to find an optimal solution to a planning or decision problem; (2) to answer a variety of what-if questions; (3) to establish understandings of the relationships among the input data items within a model; and (4) to attempt to extrapolate past data to derive meaning.

Wikipedia
A mathematical model is an abstract model that uses mathematical language to describe the behaviour of a system. Mathematical models are used particularly in the natural sciences and engineering disciplines (such as physics, biology, and electrical engineering) but also in the social sciences (such as economics, sociology and political science); physicists, engineers, computer scientists, and economists use mathematical models most extensively.

Today we might:

a. Explore one or more of these type of modeling problems
b. Think about the math learning a person "gets" from that activity
c. Consider the kind of questions that are good to ask students while they are doing these problems
d. Decide whether the problem we did together can go directly to our classrooms or whether it might need to be adapted.

30 m. 2. Working on the Problem Ourselves.
Distribute Handout 4C—Part 1, pp. 24–26. Ask the group to decide on one problem they would like to explore, then have people work in pairs on the problem. Time varies with the problem chosen. All these problems provide opportunities to link the multiple representations of the functional relations. Questions should **always** push to link the features of the graph (line of best fit) with the features of the equation (y = mx + b) and with aspects of the situation. For example, "In your equation, **what does the 'm' represent in** the graph? What does it represent in the situation?" and "In your equation, **what does the 'b' represent in** the graph? What does it represent in the situation?"

While these problems can all be made more challenging by extending to statistics (regression, correlation, etc.), they are included in this bundle to reinforce the algebraic idea of using a function to estimate real data, and examining the algebraic properties of the function.

Bring the groups back together and have them share what they discovered as they explored.

30 m. 3. Thinking About Teaching.
Distribute Handout 4C—Part 2: Using Modeling Data Problems, p 27. Have people work in small groups to respond to the questions, then bring folks together to share their thoughts.

Handout 4C—Part 1

Oil Changes and Engine Repairs

The table below displays data that relate the number or oil changes per year and the cost of engine repairs. This activity uses these data to introduce students to modeling with a linear function. To predict the cost of repairs from the number of oil changes, use the number of oil changes as the *x* variable and engine-repair cost as the *y* variable.

When graphing the data, it is important to ask students how the axes should be labeled. After a class discussion, it will be decided that the *x*-axis should be labeled "Number of Oil Changes per Year" (with a scale of 0–10), and the *y*-axis should be labeled "Engine Repair Cost (in dollars)" (with a scale of 0–700).*

Oil Changes Per Year	3	5	2	3	1	4	6	4	3	2	0	10	7
Cost of Repairs ($)	300	300	500	400	700	400	100	250	450	650	600	0	150

Oil Changes and Engine Repair

Use the Line of Best Fit Tool (http://illuminations.nctm.org/ActivityDetail.aspx?ID=146) to graph the data for the class. Project the graph onto the overhead projector or computer/television monitor. (Alternatively, students may graph the data using grid paper and pencil.)

*We suggest that you change this to a more open discussion that lets the class decide the scale and then discuss why.

Handout 4C—Part 1

Weights and Drug Doses

The dosage chart below was prepared by a drug company for doctors who prescribed Tobramycin, a drug that combats serious bacterial infections such as those in the central nervous system, for life-threatening situations.

Weight (pounds)	Usual Dosage (mg)	Maximum Dosage (mg)
88	40	66
99	45	75
110	50	83
121	55	91
132	60	100
143	65	108
154	70	116
165	75	125
176	80	133
187	85	141
198	90	150
209	95	158

1. Use grid paper to plot the data (weight, usual dosage) and draw a best-fit line.
2. Plot (weight, maximum dosage) on the same axes. Draw a best-fit line.
3. Find the slope for each line. What do they mean, and how do they compare?
4. Write the equations of the two lines.
5. Are the two lines parallel? Why or why not?
6. Use a graphing calculator to plot (usual dosage, maximum dosage). Use the calculator to construct a regression line for this data set. How does this line compare to the two lines found in 1 and 2?

Handout 4C—Part 1

A Functional Housing Market

Alicia Womick adapted this WebQuest from a project developed by Susan Boone. Alicia's version examines the Spartanburg housing market, whereas the original looked at the Houston, Texas housing market. The activity is found online at:

http://www.spa3.k12.sc.us/house.html

Purpose: Students will access the Internet to search for housing prices in Spartanburg, South Carolina, (the location can be changed to accommodate any location) and compare the prices to the number of square feet found in the living area of a house. A linear equation will be derived from these data on a coordinate plane. Any "best- fit" method for determining the graph of the line can be used. Using information from the graph of the data and the equations of the function, students will answer questions about housing prices.

Materials: Internet connection, graph paper, and ruler.

Prior knowledge: Students should be able to plot points on a coordinate plane and write an equation in slope-intercept form from a linear graph.

Description: Often, actual data do not represent an actual linear function. Students will be asked to access data from the Internet and derive a linear regression from their set of data. These data will be used to answer questions on average cost per square foot, land values, and to predict the cost of various sized homes.

Handout 4C—Part 2

Using Modeling Data Problems in Your Classroom

1. Think about the problem you just worked on. What math learning do people get from doing this kind of problem?
2. Teachers who ask good questions while students work on these kinds of problems help students get the most "mathematical juice" out of the problems. The following are a list of questions suggested in the book *Fostering Algebraic Thinking* by Mark Driscoll. Which of these questions do you think are worthwhile for your students?

Questions for Building Rules to Represent Functions
Is there a rule or relationship here?
How does the rule work, and how is it helpful?
Why does the rule work the way it does?
How are things changing?
Is there information here that lets me predict what's going to happen?
Does my rule work for all cases?
What steps am I doing over and over?
Can I write down a mechanical rule that will do this job once and for all?
How can I describe the steps without using specific inputs?
When I do the same thing with different numbers, what still holds true? What changes?
Now that I have an equation, how do the numbers (parameters) in the equation relate to the problem context?

Can you think of other questions that would be useful?

3. Would you consider using the problem "as is" in your classroom? Why or why not? What adaptations, if any, would you make?

Algebraic Thinking
Connecting to State Standards

The National Council of Teachers of Mathematics has emphasized that students should have opportunities to develop algebraic thinking at all levels.

> Instructional programs from pre-kindergarten through grade 12 should enable all students to:
>
> - understand patterns, relations, and functions;
> - represent and analyze mathematical situations and structures using algebraic symbols;
> - use mathematical models to represent and understand quantitative relationships;
> - analyze change in various contexts.
>
> *Principles and Standards for School Mathematics* (2002), NCTM p. 37.

Check out the algebra portion of your state's ABE Math Standards.

1. Is algebra included at all levels, as NCTM suggests?
2. What aspects of algebra are included?
   Are multiple representations included at all levels?
   Are patterns functions and relations included at all levels?
   What aspects of algebraic symbol sense are developed?
3. How does the treatment of each aspect of algebra differ from level to level?
4. Do you think the algebraic emphasis at each level is correct? Are there any revisions you would suggest to strengthen your state standard's treatment of algebra?
5. Check out another TIAN state's standards listed in the links below—do you notice any differences?

Below are links to each of the 6 states standards web pages.

Arizona

http://www.ade.az.gov/adult-ed/adult_ed_standards.asp

Kansas

http://adultnumeracy.terc.edu/pdfs/KS_state_standards.pdf

Louisiana

http://www.doa.louisiana.gov/osr/lac/28v129/28v129.doc.

Massachusetts

http://www.doe.mass.edu/acls/frameworks/

Ohio

http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&TopicRelationID=966&Content=21875

Rhode Island

http://www.brown.edu/lrri/standards.html,

Articles and References (For Teachers) About Algebraic Thinking

Read and discuss some of these articles with fellow teachers. Use the attached Research Reading Response (p. 32) to help guide your discussions. Please share your thoughts with others by posting online to the TIAN Talk discussion list at tian-talk@cls.coe.utk.edu.

Driscoll, Mark and John Moyer. "Using Students' Work as a Lens on Algebraic Thinking" in *Mathematics Teaching in the Middle School*, 6:5 (January 2001): 282 – 287. http://my.nctm.org/eresources/mtms/focus_mtms04.asp. This article focuses on the use of student work to help teachers focus on teaching and learning algebra.

Kalchman, Mindy S. "Walking through Space: A New Approach for Teaching Functions" in *Mathematics Teaching in the Middle School*, 11:1 (August 2005): 12–17. Kalchman shares a real-life scenario for introducing several key aspects of linear functions.

Kriegler, Shelley. "Just What is Algebraic Thinking?" Submitted for *Algebraic Concepts in the Middle School.* http://www.math.ucla.edu/~kriegler/pub/algebrat.html. Kriegler discusses three lenses for looking at algebra: algebra as abstract arithmetic, algebra as language, and algebra as a tool for the study of functions and mathematical modeling.

Mooney, Edward S. "Cookies" in *Mathematics Teaching in the Middle School*, 12:7 (March 2007): 374–377. This brief article gives you insight into students' mathematical thinking about this problem: Tim ate 100 cookies in 5 days. Each day he ate 6 ore than the day before. How many cookies did he eat on the first day?

Mooney, Edward S. "Elizabeth's Long Walk" in *Mathematics Teaching in the Middle School*, 12:5 (December 2006): 263–265. This brief article gives you insight into students' mathematical thinking about this problem: Elizabeth visits her friend Andrew and then returns home by the same route. She always walks 2 km/h when going uphill, 6 km/h when going downhill, and 3 km/h when on level ground. If her total walking time is 6 hours, then what is the total distance she walks?

Peterson, Blake E. "Counting Dots and Measuring Area: Rich Problems from Japan" in *Mathematics Teaching in the Middle School*, 12:4 (November 2006): 214–219. Students look at a set of dots and create a variety of generalizable patterns.

Smith, Margaret S., Amy F. Hillen, and Christy L. Catania. "Using Pattern Tasks to Develop Mathematical Understandings and Set Classroom Norms" in *Mathematics Teaching in the Middle School*, 13:1 (August 2007): 38–44. This article discusses the use of pattern blocks to help students develop algebraic reasoning and to establish classroom norms and practices.

Thomas, David A. and Rex A. Thomas. "Discovery Algebra: Graphing Linear Equations" in The Mathematics Teacher, 92:7 (October 1999): 569 – 572. http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MT1999-10-569a. In this article, Thomas shares his personal classroom experience moving toward a new approach to teaching algebra.

Additional Resources

Driscoll, Mark. (1999). *Fostering Algebraic Thinking: A Guide for Teachers,* Grades 6–10. Portsmouth NH: Heinemann.

Research Reading Response

Title/Author/Reference:

Main Ideas:

- What were the author's main ideas regarding adults/children *learning* mathematics?

- What were the author's main ideas regarding *teaching* mathematics to adults/children?

Applications:

- What are the implications for you as an ABE mathematics teacher?

Classroom Resources
To Strengthen Algebraic Thinking

This is a starter list of classroom resources that focus on algebraic thinking using models and real-life investigations. If you know of other resources, please share with others by posting online to the TIAN Talk Discussion list at tian-talk@cls.coe.utk.edu.

Published and Online Resources

NCTM Illuminations Lessons

Illuminations is packed with activities and lessons for grades K–12, many of which can be adapted for adult learners. The lesson "Line of Best Fit" (http://illuminations.nctm.org/ActivityDetail.aspx?ID=146), gives students an opportunity to explore lines of best fit using an applet. "Impact of a Superstar" (http://illuminations.nctm.org/LessonDetail.aspx?ID=L673) uses the line of best fit to compare two basketball teams.

NOVA Online

NOVA has quite a few interesting activities that integrate science, social studies, and math. The activity "Supersonic Dream" (http://www.pbs.org/wgbh/nova/teachers/activities/3203_concorde.html) has students comparing the fuel consumption and per person fuel cost of seven different aircraft.

EMPower Books

Most of the lessons in *EMPower's Seeking Pattern, Building Rules: Algebraic Thinking* provide opportunities to use multiple representations to model a situation.
http://www.keypress.com/x5153.xml

Discovering Algebra

Tying Knots and *Beam Strength* are activities in Discovering Algebra that address modeling data and using real-world data. The chapter that includes the *Tying Knots* activity is available as a free PDF download if you click on the link to Chapter 3 at:
http://www.keypress.com/x5265.xml

Finding functions to model the data

http://www.northcanton.sparcc.org/~technology/excel/files/scatterplots.html

Ideas

All Chocolate, No Change

In this article from NCTM, students explore different situations in which they have to determine the number of chocolate bars they can buy without having leftover change. After determining the pattern for each pair of chocolate bars, students then graph the resulting equations.
http://my.nctm.org/eresources/article_summary.asp?URI=MTMS2006-02-262a

Bungee M&Ms

This activity, which also includes student and teacher notes, asks students to explore the idea of bungee jumping using M&M'S®. Students graph the data as they test out different scenarios.
http://fcit.usf.edu/FCAT8m/resource/activity/bungeet.htm

Female Frontiers

This is an entire unit of activities from NASA. Included in the unit is a discussion of scatterplots and lines of best fits with student activities.
http://quest.nasa.gov/space/frontiers/activities/aeronautics/m.html

Get a Half Life

This activity, complete with student handouts and teacher notes, has students exploring a constant rate of change that is not linear. Students create a "curve" of best fit rather than a line of best fit. http://fcit.usf.edu/FCAT8m/resource/activity/halflift.htm

Scatter Plot Explorations

http://exploringdata.cqu.edu.au/sctrplot.htm is a nice website that includes several activities that teachers can do with their students. One of the activities: http://exploringdata.cqu.edu.au/ws_scatr.htm looks at actual data from the Challenger. Create a scatter plot from the data and decide what should have been done. The activity from http://exploringdata.cqu.edu.au/movies.htm has students collecting their own data and then plotting the points and looking for relationships.

http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=270 uses Gizmo for an interactive exploration with scatter plots

http://argyll.epsb.ca/jreed/math9/strand4/scatterPlot.htm allows students to plot points and watch the line of best fit change as points are added

Incompatible Math Goals?

Problem Statement

In TIAN, we emphasize understanding over procedures, and encourage students to problem-solve with a variety of methods. Some tests respect that idea, but some are focused on the computational fluency. As a teacher, I want my students to have a deeper understanding of the math, but I also want them to reach their goals—and there's so little time!

Over the course of the TIAN institutes, this was an issue that came up rather often. The following activity intends to get the discussion going among your local or regional teacher group.

1. Check all that apply.

As an adult ed **math** teacher, I prepare my students to do well on math needed for success on/in:

- ☐ State-mandated ABE progress tests such as CASAS or TABE
- ☐ State-mandated high school high stakes tests
- ☐ The GED
- ☐ Another HS diploma test (e.g. Adult Diploma, External Diploma)
- ☐ Community College Placement Test (e.g., the Accuplacer, Compass)
- ☐ College courses
- ☐ Life
- ☐ Work
- ☐ Other ________________

2. From the list of goals above, think about those that your students are focused on, and arrange them from easiest to most difficult.

3. Which of the goals emphasize problem-solving and reasoning? Which emphasize procedural knowledge?

4. So, what's the solution to the dilemma?

Introduction to the Facilitator's Guide

Each TIAN Bundle's third section (the Facilitator's Guide) is designed to give some practical suggestions about how to facilitate a teacher meeting using the resources in the other two bundle sections (Math Topic and Teaching/Learning Issue). There is a suggested Meeting Feedback Form for the group and a Teacher Meeting Notes form to submit to the tian-talk discussion list by sending an email to tian-talk@cls.coe.utk.edu. Please note these are only suggestions. The TIAN team is interested in hearing what groups decide is most important and helpful for them.

Suggestions for Using Bundle #4 in Teacher Meetings

As you plan to use a Bundle, print out a copy of the entire Bundle (about 40 pages). Read through it, deciding which sections to photocopy for the meeting and which to let group participants access themselves on the TIAN website at http://adultnumeracy.terc.eu/TIAN_teacher_resources.html

If your group has ONE two-hour meeting to spend on Bundle #4, set aside at least 2/3 of the time for the Math Topic and 1/3 of the time on the Teaching/Learning Issue or discussing one or more of the articles from the Articles and References for Teachers. So, a meeting might go something like:

1. **Introduce the Math Topic, Algebraic Thinking, either by**
   a. summarizing the main points in the Introduction or
   b. emailing the introduction ahead of time to the group members, and then briefly discuss the main points in the meeting.
2. **Do some math together.**

   This Bundle has three sets of activities. Choose one set of activities and do all the different problems. Or, you may have small groups each focus on one of the activities from each of the three different sets, then have them share what they learned. If you do this, you'll want to be sure to save time to look across the different activities for similarities and differences.

3. **Consider the issue: "Incompatible Math Goals" or discuss how effectively your state's standards address the broader definition of algebraic thinking.**

4. **Get some feedback on the meeting and ask a volunteer to send an email to tian-talk to share good ideas that came up in the meeting. Also, ask everyone to bring back to the next meeting what they did with these activities in their classes.**

If your group has TWO two-hour meetings (4 hrs) to use Bundle #4, you might spend the entire first meeting on the Math Topic, and the second meeting discussing how things played out in class, ending that second meeting with a discussion of the Teaching/Learning Issue.

In the first meeting, you might have time to do and reflect upon all three sets of activities, and to begin to choose some articles to read before the next meeting. You might start the second meeting with everyone sharing their feedback based on the article(s) they read.

The Importance of Promoting Teacher Mathematical Learning

ABE math teacher groups get together for two main reasons—to get some good math teaching ideas and resources for their classrooms and to expand their own math knowledge. The activities that you do together begin with teachers wrestling with the problems themselves. As they struggle, some things you do as facilitator will be more likely to promote mathematical learning than others. All facilitators should keep these five important ideas in mind:

1. TIAN teachers value sharing solutions among themselves and encourage sharing in the classroom. When asking people to share, encourage people to explain their thought processes.
2. In the TIAN institutes we were always interested in more than one strategy, and whether we could see the connections between the strategies.
3. Regard confusion and error as learning opportunities—don't avoid it.
4. Raise honest questions that push on the math. This means it is ok to not have the answer to the questions posed. All of us are learners—that includes the facilitator.
5. It's a community—everyone should take responsibility for the learning.

These ideas, so beautifully presented in the table on the next page, would be good for everyone in the group to have a copy of right from the first meeting.

Continuum of Sociomathematical Norms

	Less likely to promote mathematical learning ⟵	⟶	More likely to promote mathematical learning
Sharing	Ideas and solutions are shared with minimal or no explanation	Thinking is described, often in procedural terms	Explanations consist of a mathematical argument
Solution Strategies	Emphasis is on one single solution or strategy	Multiple strategies and solutions are described	Emphasis is placed on the relationships among multiple solutions and/or strategies
Confusion & Error	Confusion and mistakes are avoided or ignored, or are corrected by the PD leader	Confusion and mistakes are acknowledged in hopes of causing disequilibrium and change in understanding	Confusion and errors are embraced as opportunities to compare ideas, re-conceptualize problems, explore contradictions in solutions, or pursue alternative strategies
Questioning	The PD leader asks questions aimed at maintaining social order or eliciting specific responses	Both the PD leader and teachers raise procedural and/or factual questions about the mathematics	Both the PD leader and teachers raise questions that push on understanding of mathematics/ mathematical reasoning
Community	Work is generally done individually or ideas are shared through PD leader explication	teachers collaborate to find solutions to problems	Mathematical argumentation forms the basis of a generative learning process where individuals take responsibility for their own and the group's progress

(Adapted from Yackel & Cobb)

Meeting Feedback Form

(for the group and the facilitator)

What was the most effective part of the meeting today, and why?

What would you change for the next time? Why?

What pressing issues/topics would be good to address?

Teacher Meeting Notes

(To share with other groups on the tian-talk discussion list at tian-talk@cls.coe.utk.edu)

Date/time of meeting:

Group Title and meeting location (City or town, State)

Facilitator(s)

Number of participants present

Describe what occurred at the meeting

Did you use any activities or discuss the issue from the TIAN Bundles? How effective were the activities or discussion of the issue?

Did your group use resources others than those in the TIAN Bundles? If so, please describe (or attach).