TEACHERS INVESTIGATING ADULT NUMERACY

Bundle #3

Contents

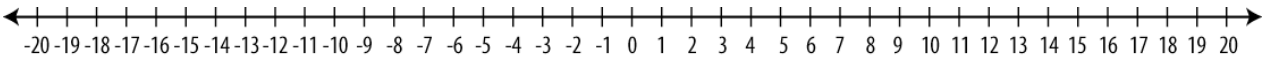
NUMBER SENSE: INTEGERS

Introduction

Minus times minus results in a plus,
The reason for this, we needn't discuss.
—Ogden Nash

The activities and resources in this bundle center around the operations with integers, or signed numbers, a topic that is often hard for students to comprehend. In the past, school lessons with positive and negative numbers often consisted of learning a set of rules. These rules are easy to learn, but even easier to forget, mix up or misapply ("like signs, positive" often lead to solutions such as (–4) + (–6) = +10). You might say the **first main message of this bundle is that just learning the rules does not provide sufficient opportunity for anyone to understand the meaning of the operations on integers.**

There are alternative approaches to introducing and working with positive and negative amounts. One approach employs the **number line** as a visual thinking tool and its "real life" metaphors, for example, temperature change, checking account, a football field, sea level position, or elevator movement. This lets us look at numbers directionally.



A second approach employs a model where positives cancel out negatives and often use tools such as **two-color chips** with a metaphor for positive and negative charges on particles, where a plus 1 and a minus 1 "zero out". This approach looks at numbers as opposites.

While teachers have found that these metaphors help students work more easily with the numbers and operations, there are situations in which each of these models gets confusing rather than illuminating. **The second main message of this bundle is that teachers need to be aware of the strengths and weaknesses of each metaphor, and be careful of mixing metaphors.** When using models, we need to be consistent, or at least aware of when we've changed our model.

The following quote from *Classroom Learning Session 10: Organizing What We Know: The Structure of the Disciplines* (Stanford University School of Education) by Linda Darling-Hammond, Suzanne Orcutt, and Kim Austin) (http://64.233.169.104/search?q=cache:st4lslZEspMJ:www.learner.org/channel/courses/learningclassroom/support/10_arch_know.pdf+dilemmas+in+teaching+negative+numbers&hl=en&ct=clnk&cd=52&gl=us) guided the creation of the four activities in TIAN Bundle #3:

> *"For certain key concepts that are always hard for students to learn, teachers need to discover, use, and think of multiple ways of approaching those concepts. This is not the same as creating a prescriptive set of activities. Rather, this approach requires ascertaining what the students already understand, or misunderstand, and applying a set of strategies to build bridges between students and the content".*

The first activity starts with student experiences. **Activity 3A: Negative Experiences** seeks to ascertain how students conceptualize negative and positive numbers as well as getting them to think about various ways to get to –50 on the thermometer/number line. After you, as teachers, try the activity, we hope you'll try it in your classrooms.

The other three activities are meant for teachers. We hope they help you begin some explorations and interesting conversations among your peers.

Activity 3B: Exploring Signed Number Models asks you to choose among three models and solve some problems with that model, then critique the strengths and weaknesses of that model.

Activity 3C: Analyzing Teaching Signed Numbers sends your group to check out two teachers who have posted instructional videos on the web. We'd like you to critique the instructional decisions.

Activity 3D: Assessing Student Thinking asks you to assess student learning using the DEN process (**d**escribe, **e**valuate, **n**ext steps) from two classroom transcripts.

Last but not least, please contribute ideas and resources that have worked for you and other teachers might make use of.

Have fun!

Number Sense: Integers

Activity 3A. Negative Experiences

Goal: To have participants make connections between negative numbers and real-life contexts by generating examples of negative numbers they have come across in their lives. To understand how they "see" a negative amount.

Time estimate:

45 minutes

Focus:

Number Sense: Integers

Materials:

✓ *Negative Experiences Handout* (p. 5)

Preparation:

Try the activity yourself. How many negative number, real-life connections occur to you? How many ways can you "see" minus 50? And how many ways can you think of for getting to –50?

This activity is designed for teachers to explore together before doing something similar with their students. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE FROMYOUR TEACHER GROUP OR STUDENTS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

Part 1. What do negatives look like?

5 m. **1.** Introduce the topic of signed numbers by writing "-50" on the board and asking: "How do you say this in words?" (People might say 50 in the hole, minus 50, negative 50, 50 below zero, etc.) Then

probe, "Where have you seen this number in your own life?" After giving everyone think-time, ask someone to share an example.

15 m. 2. Distribute the *Negative Experiences Handout*. Ask individuals to record examples of negative numbers from their own personal experiences, how they might visualize those numbers based on their own examples, as well as the visual representation of the positive counterpart.

When everyone has had a chance to think of at least two examples, ask individuals to share their ideas in small groups.

10 m. 3. Bring the groups back together and have them share what they discovered and ask for any particularly interesting situations in which they saw signed numbers. Then ask:

- How might this activity play out with your own students?
- What do you predict students will say when you ask for examples of real life use and pictures of "-50"? Try it and see if your predictions play out. Be cognizant of you own "wait time" to let students' ideas emerge.

For facilitators/teachers to think about

Remember that the point is for participants to first focus on their own personal experiences, not their students'.

Part 2. How many ways to get to –50?

15 m. Use one of the metaphors/real-life contexts people have offered that is most like a number line (e.g., thermometer, elevator, sea level) and sketch it or use the one on pp. 10–12.

Ask: What are some ways to get to -50 degrees?

This question (similar to number of the day) has an infinite number of correct answers and is meant to get people thinking about the number line's direction and distance. Ask people to trace the action on the number line, say it in words, and write the notation.

5 m. Ask:
How would you bring this to class? What do you think the value of this question is? Where could it lead to in order to build understanding?

Negative Experiences Handout

Think about where you have seen or used negative numbers in your own life. Give two specific examples of where you see them, how you would picture or visualize them, and how you might compare them to their positive counterpart.

First example of negative numbers in my life	
Picture that helps me "see" the value	**What a similar positive amount looks like**

Second example of negative numbers in my life	
Picture that helps me "see" the value	**What a similar positive amount looks like**

Number Sense: Integers

Activity 3B. Exploring Signed Number Models

Goal: For teachers to explore and critique various visual models used to teach and learn about signed numbers

Time estimate:

50 – 60 minutes

Focus:

Number Sense: The four operations with integers

Materials:

- ✓ Red and yellow chips (or other two-color counters; http://www.etacuisenaire.com/catalog/product?deptId=&prodId=4060&q=two-color+counters) **or** Positive/Negative Squares (p. 10)
- ✓ Thermometers (p. 11)
- ✓ Number lines (p. 12)
- ✓ Play money (p. 13)
- ✓ *Exploring Integer Models Handout* (p. 9)

Preparation

Try out each model so there are no surprises in class. Read Dr. Math discussion at Ask Dr. Math FAQ from The Math Forum @ Drexel. http://mathforum.org/dr.math/faq/faq.negxneg.html.

This activity is designed for teachers to explore together. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE IN TEACHER DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

If you have access to the internet, the following links may be used in place of physical manipulatives and number lines.

The virtual manipulatives applet (from the National Library of Virtual Manipulatives at Utah State University) uses plus-minus chips to demonstrate adding and subtracting positive and negative values. Instructions prompt you along to find your answer.

http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html

http://nlvm.usu.edu/en/nav/frames_asid_162_g_2_t_1.html

A virtual number line applet can be found at the following web site:
http://www.mste.uiuc.edu/java/michael/numberline/numberline.html

Click on the number line to activate.

You'll find a vast array of math/science applets can be found on this web site:
http://www.explorelearning.com/index.cfm?method+cResource.dspDetail&ResourceID=95

Suggested Activity Sequence:

10 m. **1.** Introduce the topic of signed numbers and ask participants to think about the following:

"What's the hardest thing to teach/learn about signed numbers?"

Once individuals have shared some of the challenges, explain that they are going to explore different methods to teaching/learning about operation on signed numbers.

25 m. **2.** Distribute the *Exploring Integer Models Handout*. Ask participants to choose one of three models to explore operations with integers.

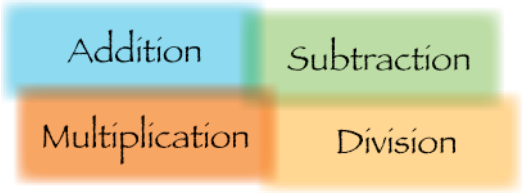
a. two-color chips or positive/negative squares

b. number line or thermometer

c. money/banking

Have participants form small groups. Make sure that each model is chosen by at least one group. Explain that they are to try to use the model to show all four operations and use the model to solve the problems on the handout.

Explain that they are to determine how the model does/does not work well for:



If a group has fully explored one model, encourage them to move to another.

10 m. **3.** Bring the groups back together and have them share what they discovered and have them present how their model works. Be sure each model is discussed as to its pluses and minuses for each operation.

10 m. **4.** Once each model has been discussed, ask participants if they have other strategies for teaching signed numbers. Allow participants to share their strategies. If no one mentions it, you might want to show them the pattern strategy which "proves" two negatives multiplied together equals a positive number.

For facilitators/teachers to think about

Remember to first model and then LISTEN and OBSERVE as participants do the activity themselves. Be explicit when you model since you hope that they will do likewise with their own students.

Exploring Integer Models Handout

The manipulative or visual model chosen to solve these problems is

_____.

Keep track. Put a check by the problems you can successfully solve with the model, and circle the problems that are confusing or unwieldy with the model.

Add.

4 + 7 =	7 + -1 =
5 + -5 =	-3 + -2 =
-5 + 3 =	0 + -8 =

Subtract.

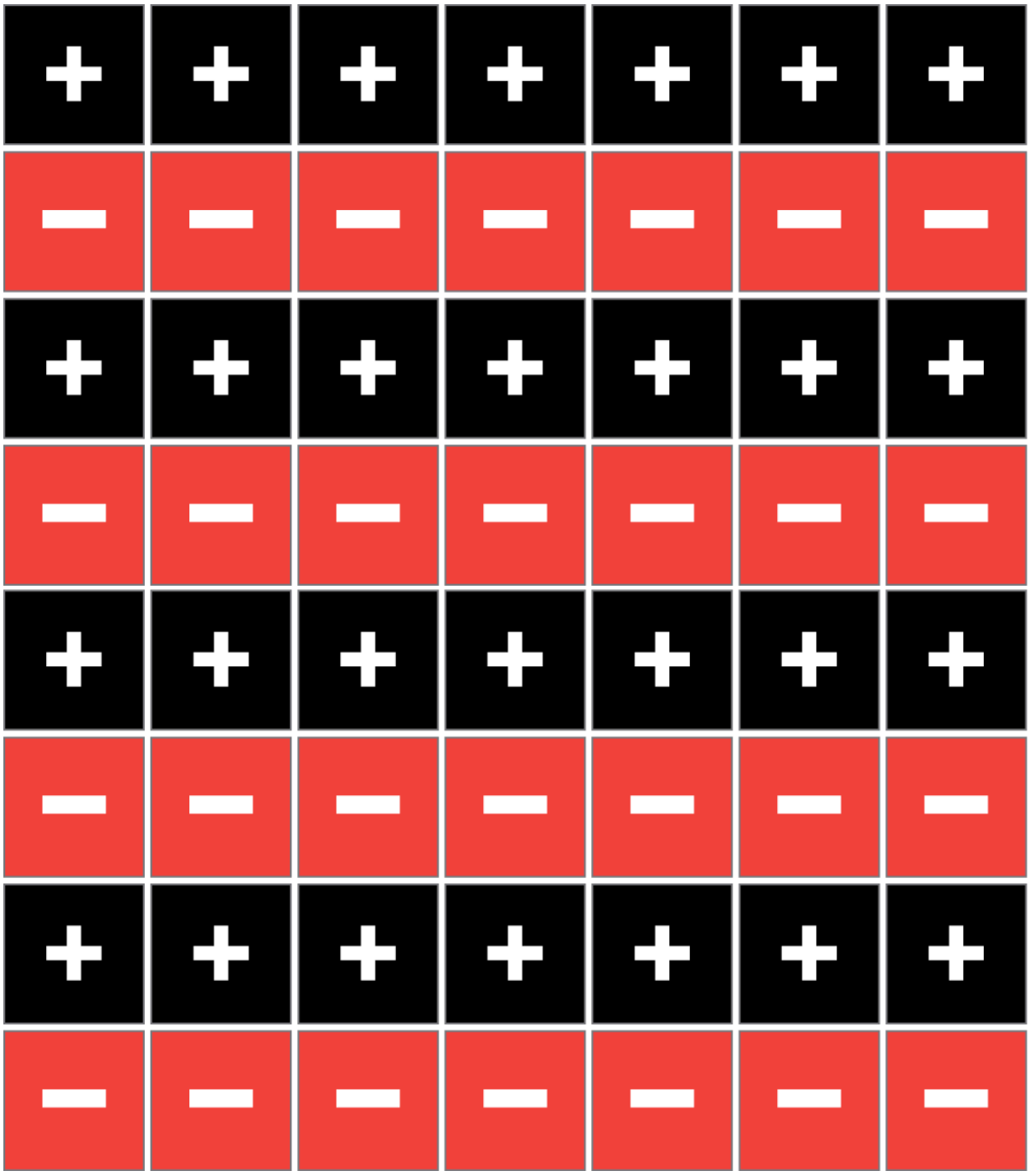
4 – 7 =	7 – -1 =
5 – (-5) =	3 – -2 =
-5 – 3 =	0 – -8 =

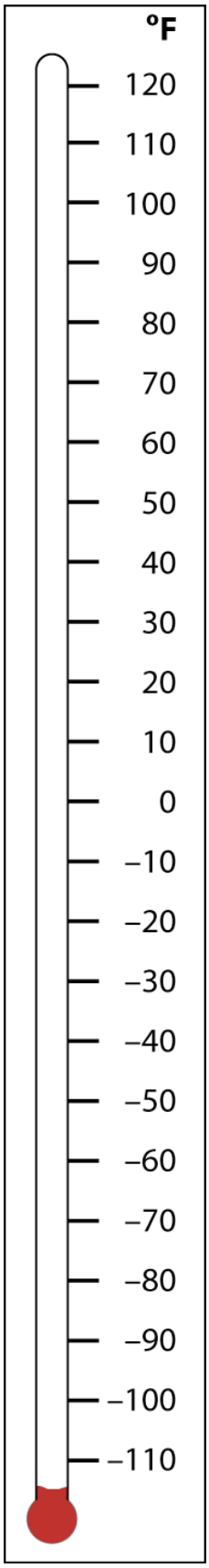
Multiply.

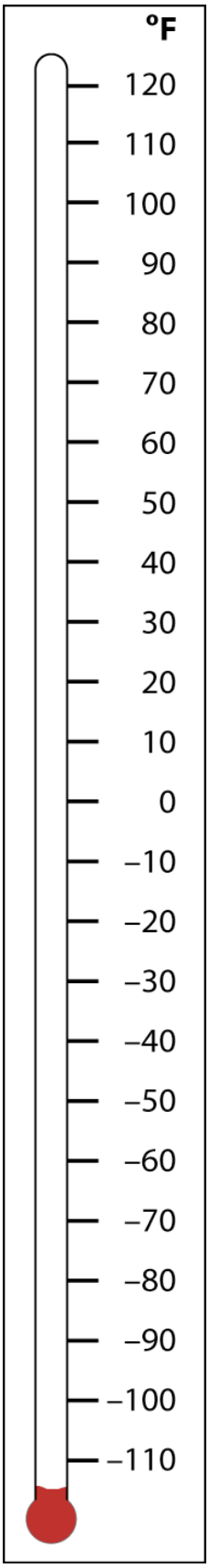
4 * 7 =	7(-1) =
5(-5) =	-3(-2) =
-5(3) =	0(-8) =

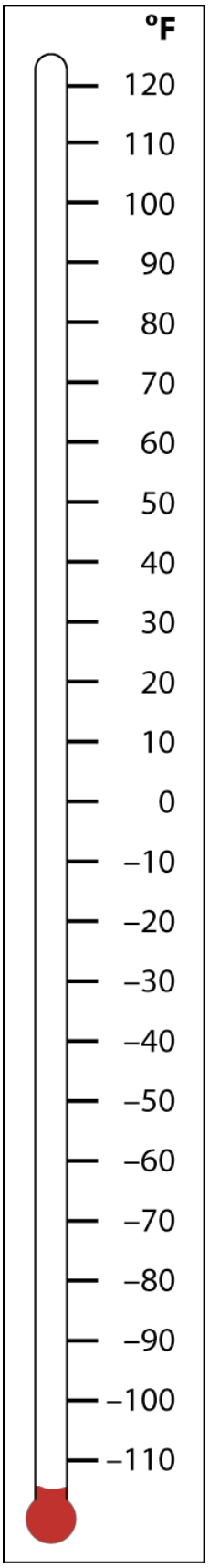
Divide.

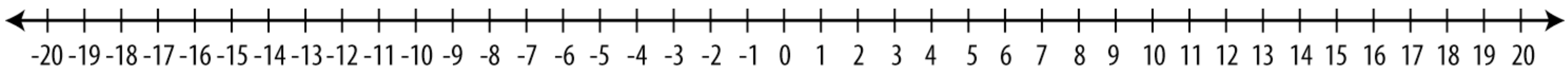
28/7=	7/-1 =
5/-5 =	-6/-2 =
-15/3 =	0/-8 =

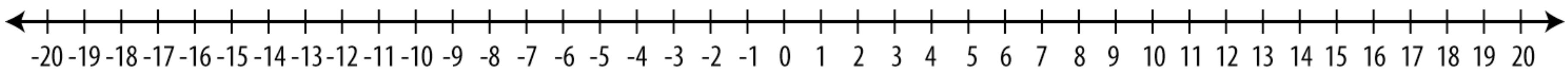


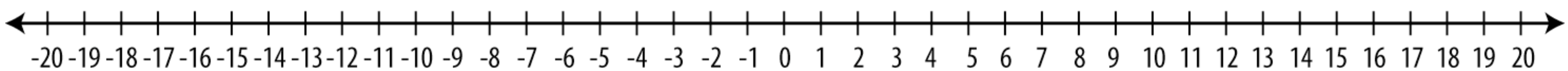


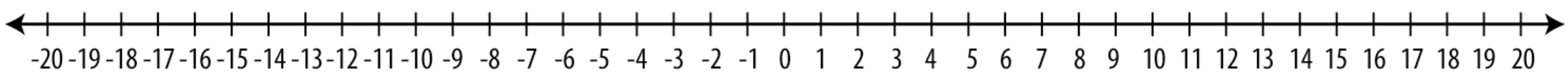


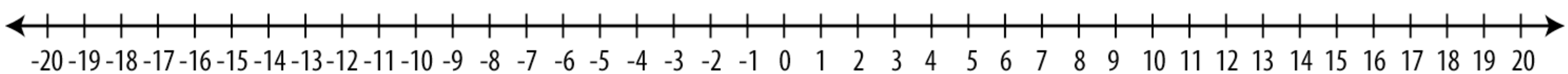


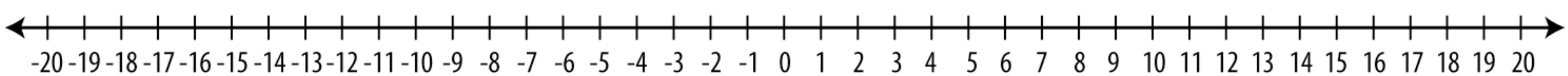


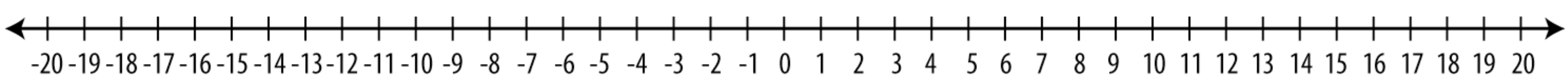


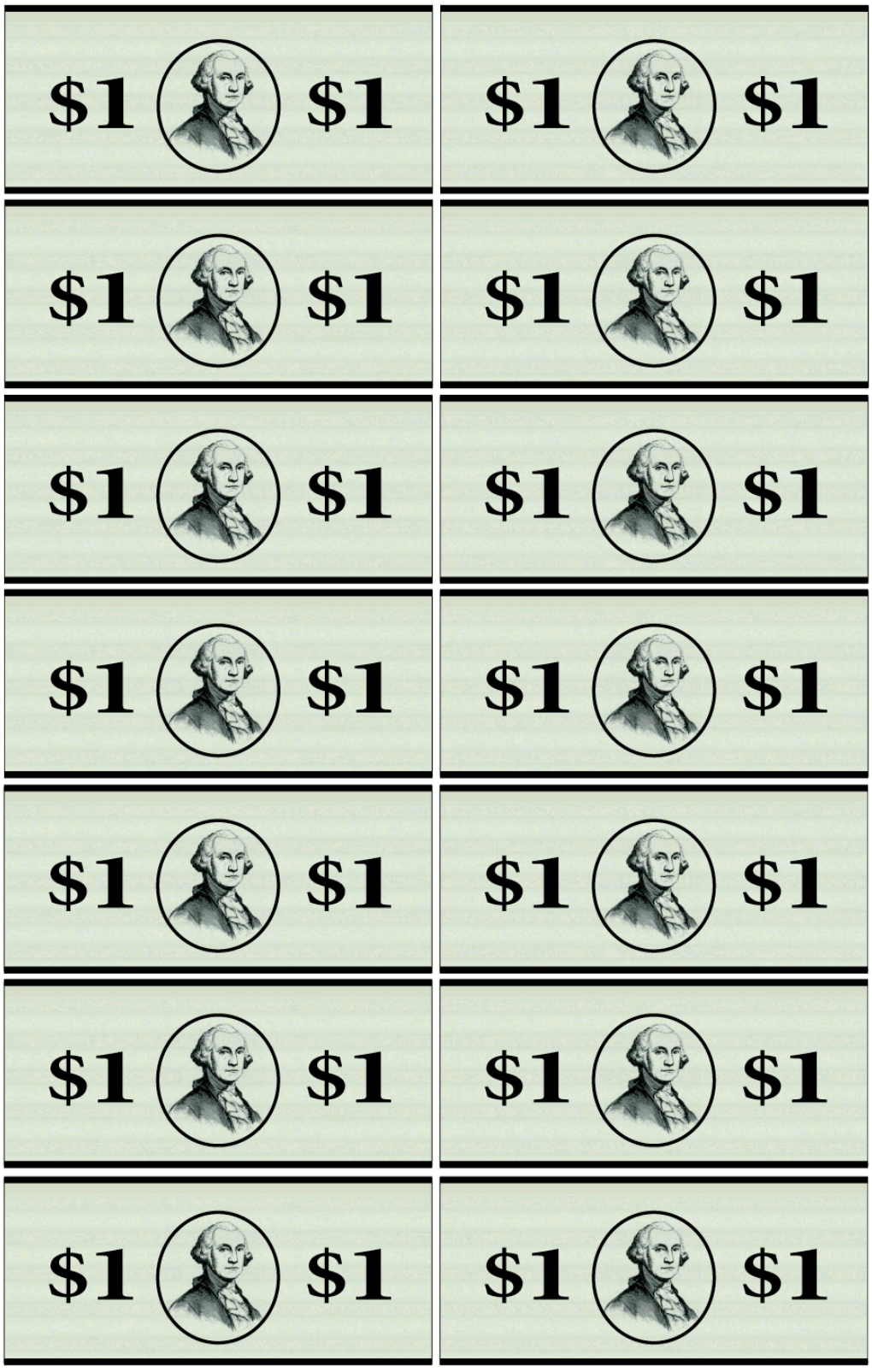












Number Sense: Integers

Activity 3C. Analyzing Teaching Signed Numbers

Goal: To critically analyze how signed numbers are introduced to students using video examples

Time estimate:
30 - 40 minutes

Focus:
Number Sense: The four operations with integers

Materials:
✓ *Analyzing Models Handout* (page 16)

Preparation
Set up video ahead of time and be sure technology is working.
Preview video(s) to be used. Suggested video: http://www.youtube.com/watch?v=C38B33ZywWs or http://www.youtube.com/watch?v=YgLk7UFEAb8, both which begin with the number line model

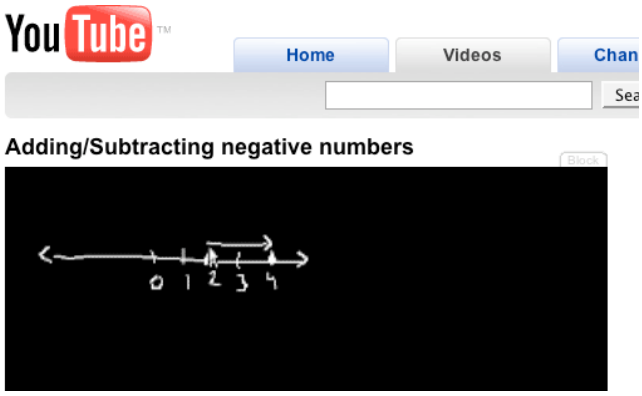
This activity is designed for teachers to explore together. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE IN TEACHER DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

5 m. **1.** Explain the value of using visual/physical models such as the number line, a thermometer, banking, or two-color chips to teach big ideas in math, which includes signed numbers. The models need to be consistently used, or students can become confused. While we as teachers sometimes think we're doing a great job of explaining an idea to our students, we don't always realize when we've shifted our discussion from one model to another.

Explain that this activity will give participants an opportunity to look critically at an explanation of some aspect of signed numbers.

15 m. **2.** Distribute the *Analyzing Models Handout*. Review it with participants before showing the video. Ask participants to use the handout as a guide as they watch an instructor teach about signed numbers.



Show the video. Once the video is done, give participants another 5 minutes to write their thoughts.

15 m. **3.** Bring participants back together and have them share their thoughts. Then ask: "How might this activity make you more aware of your own teaching?"

For facilitators/teachers to think about

If participants do not see any change in models, you may want to have them watch the video again, pausing when the instructor switches from one model to another.

Analyzing Models Handout

As you watch the video of an instructor teaching a lesson on signed numbers, think about the following questions:

1. What you think the instructor did well?

2. What assumptions did the instructor make?

3. Where did the explanation get potentially confusing? Did the instructor ever mix models?

4. Based on what you viewed, how might you change how you teach signed numbers?

Number Sense: Integers
Activity 3D. Assessing Student Thinking

Goal: To probe students' thinking about negative numbers

Time estimate:
40 - 45 minutes

Focus:
Number Sense: Integers

Materials:
- ✓ *Vignettes Handout* (pp. 19–20)
- ✓ *The DEN Process Handout* (p. 21)

Preparation
Read the vignettes taken from *Learning Mathematics as Developing a Discourse* by Anna Sfard of the University of Haifa. (pp. 8 and 12) (http://mathcenter-k6.haifa.ac.il/articles(pdf)/sfard.pdf)

This activity is designed for teachers to explore together. PLEASE SHARE INTERESTING EXPERIENCES THAT ARISE IN TEACHER DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

5 m. **1.** Distribute *The DEN Process Handout*. Explain that during the TIAN Pilot and Field Test Institutes, teachers used the DEN method to analyze student's work.

The DEN Process
D = Describe what is happening
E = Evaluate what is happening; what does the student know, not know
N = Next steps: based on what student knows/does not know, decide what you would do next to help him.

25 m. **2.** Distribute the *Vignettes Handout*. Ask participants to choose one of the vignettes and then use the DEN process to analyze students' thinking.

10 m. **3.** Bring the groups back together and have them share what they discovered.

10 m. **4.** Close by asking participants about the DEN process itself: How did the process help them think about student's understanding?

For facilitators/teachers to think about

Remember to first model and then LISTEN and OBSERVE as participants do the activity themselves. Be explicit when you model since you hope that they will do likewise with their own students.

Vignettes Handout

Vignette 1:

[This vignette is taken from *Learning Mathematics as Developing a Discourse* (p. 8) by Anna Sfard of the University of Haifa (http://mathcenter-k6.haifa.ac.il/articles(pdf)/sfard.pdf)]

The teacher asked what 2·(-5) could be equal to.

[N13] Sophie: Positive two times negative five...

[N14] Ada: Two times negative five..

[N15] Sophie: Aha, hold on... hold on... It's as if you said negative five multiplied two times.... So, negative five multiplied two times it's negative ten...

[N16] Roy: Negative ten.

[N17] Teacher: Why?

[N18] Roy: We simply did... two times negative five equals negative ten because five is the bigger number, and thus... uhmm... It's like two times five is ten, but [it's] negative ten because it is negative five. .....

[N42] Noah: And if it was the positive seven instead of positive two?

[N43] Yoash: Then there will be positive thirty five.

[N44] Sophie: Why?

[N45] Yoash: Because the plus [the positive] is bigger.

Think about what you hear happening with Yoash's and Roy's thinking, then consider about what they know and don't know. Then consider what you would do if these were your students.

Vignette 2:

[This vignette is taken from *Learning Mathematics as Developing a Discourse* (p. 12) by Anna Sfard of the University of Haifa (http://mathcenter-k6.haifa.ac.il/articles(pdf)/sfard.pdf)]

Dan explains his difficulty with negative numbers

[1] Dan: Minus is something that people invented. I mean... we don't have anything in the environment to show it. I can't think about anything like that.

[2] Anna: Is everything that regards numbers invented by people?

[3] Dan: No, not everything...

[4] Anna: For instance?

[5] Dan: For example, the basic operation of addition, one plus one [is two] and according to the logic of the world this cannot be otherwise.

[6] Anna: And half plus one-third equals five sixths. Does it depend on us, humans or...

[7] Dan: Not on us. You can show it in the world.

[8] Anna: I see... and 5 minus 8 equals - 3. It's us or not us?

[9] Dan: It's us.

[10] Anna: Why?

[11] Dan: Because in our world there is no example for such a thing.

Think about what you hear happening with Dan's thinking, then consider about what he knows and doesn't know. Then consider what you would do if this were your student.

The DEN Process

D = Describe what is happening
E = Evaluate what is happening; what does the student know, not know
N = Next steps: based on what student knows/does not know, decide what you would do next to help him

Describe, in general terms, what you see happening.

For example, the student is ...

- using his fingers to multiply by 5's
- drawing a picture to show the number of boxes of cereal for $10
- adding the ones in the tens column and counting them as units rather than tens
- multiplying the number in the tens column and placing the total in the ones column

Evaluate what is happening, by considering what the students seems to know/not know.

For example, the student is able to...

- count by 5's, but doesn't seem to know his 5 facts by memory
- determine the ratio of cereal to dollars
- add correctly but doesn't seem to understand what the digits in the tens column represent
- multiply correctly—student can multiply 1(0) x 5, but the digits are in the wrong place. Doesn't seem to understand that the one in the tens column represents ten, not one

Decide Next Steps, based on your evaluation.

For example, you might...

- ask the student to start looking for patterns in the five table (even number answers end in zero, odd in five).Practice facts out of order.
- have student continue the pattern of drawing, then have him replace the drawings with numbers to see if he can still see the pattern with only numbers and no drawings
- have students practice with actual manipulatives (i.e., popsicle sticks) and bundle groups of ten with a rubber band so that they can see that one in the tens place is really one bundle of ten.
- have students practice with actual manipulatives (i.e., popsicle sticks) and bundle groups of ten with a rubber band so that they can see that one in the tens place is really one bundle of ten, and having five of those one-bundle sets is a total of 50, not five.

Number Sense: Integers

Connecting to State Standards

How and where (at which levels and under which math topics) do your state math standards or benchmarks address the topic of number sense? Are multiple ways of understanding the operations, especially with visual models explicitly described? Implicitly assumed?

When is understanding of symbols such as the equal sign and the concept of equality addressed? Do you think your state standards do a good job of anticipating algebra in the arithmetic section?

Below are links to each of the 6 states standards web pages.

Arizona

http://www.ade.az.gov/adult-ed/adult_ed_standards.asp

Kansas

http://adultnumeracy.terc.edu/pdfs/KS_state_standards.pdf

Louisiana

http://www.doa.louisiana.gov/osr/lac/28v129/28v129.doc.

Massachusetts

http://www.doe.mass.edu/acls/frameworks/

Ohio

http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&TopicRelationID=966&Content=21875

Rhode Island

http://www.brown.edu/lrri/standards.html,

Articles and References (For Teachers) About Positive and Negative Integers

Minus times minus results in a plus,
The reason for this, we needn't discuss.
—Ogden Nash

Read and discuss some of these articles with fellow teachers. Use the attached Research Reading Response (p. 25) to help guide your discussions. Please share your thoughts with others by posting online to the TIAN Talk discussion list at tian-talk@cls.coe.utk.edu.

Ask Dr. Math FAQ from The Math Forum @ Drexel. http://mathforum.org/dr.math/faq/faq.negxneg.html Provides several examples of rationale for why multiplying two negatives yields a positive integer.

Baldwin, John and Cathy Kessel. "Concept and Computation: The Role of Curriculum" for MER Newsletter http://www.math.uic.edu/~jbaldwin/pub/kessel1.html. A look at how and when Chinese and American curricula handle negative numbers.

Ball, Deborah Loewenberg. "With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics" based on an earlier version presented at the April 1990 meeting of the American Educational Research Association in Boston. http://ncrtl.msu.edu/http/craftp/html/pdf/cp903.pdf. While the dilemmas Ball discusses are about issues much larger than the topic of negative numbers, pages 7–17 and pages 23–31 of the article focus specifically on examples in teaching negative numbers.

Gregg, Jeff and Diana Gregg. "A Context for Integer Computation" from *Mathematics Teaching in the Middle School*, 13:1 (August 2007): 46–50. http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MTMS2007-08-46a Suggests the context of allowance rather than debt or other "contrived" situations for bringing more meaning to integer computation.

Kent, Laura Brinker (2000). "Connecting Integers to Meaningful Contexts" in *Mathematics Teaching in the Middle School*, 6: 1: (September 2000). 62–66. http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MTMS2000-09-62a This article illustrates how students can begin to intuitively learn the rules for adding and subtracting integers.

Negative Times Negative Is What? http://www4.ncsu.edu/unity/lockers/users/f/felder/public/kenny/papers/negative.html. This site offers four explanations to the question of why multiplying two negatives gives you a positive number.

National Research Council (2001). Number: What Is There to Know?" from *Adding It Up: Helping Children Learn* (80–83, 244–246).

Nurnberger-Haag, Julie (2007). "Integers Made Easy: Just Walk It Off" in *Mathematics Teaching in the Middle School*, 13:2 (September 2007). http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MTMS2007-09-118a. This quick read offers a multisensory method to teach the rules associated with adding, subtracting, multiplying and dividing integers.

Peled, Irit and David Carraher (2006). Signed Numbers and Algebraic Thinking (pp. 303–328). (http://www.earlyalgebra.terc.edu/our_papers/2007/2007-4Peled_Carraher_Kaput_Ch_12.pdf) Researchers investigate the relationship between algebra and signed numbers and suggest that algebra provides more meaningful models for signed numbers.

Sfard, Anna. "Learning Mathematics as Developing a Discourse". http://mathcenter-k6.haifa.ac.il/articles(pdf)/sfard.pdf. While Sfard's key message is the importance of communication in learning mathematics, one of her two examples focuses on students grappling with negative numbers.

Research Reading Response

Title/Author/Reference:

Main Ideas:

- What were the author's main ideas regarding adults/children *learning* mathematics?

- What were the author's main ideas regarding *teaching* mathematics to adults/children?

Applications:

- What are the implications for you as an ABE mathematics teacher?

Classroom Resources
To Strengthen Number Sense: Integers

This is a starter list of classroom resources that focus on models for signed numbers. If you know of other resources, please share with others by posting online to the TIAN Talk Discussion list at tian-talk@cls.coe.utk.edu.

Published and Online Resources

National Library of Virtual Manipulatives

The virtual manipulatives applet (from the National Library of Virtual Manipulatives at Utah State University) uses plus-minus chips to demonstrate adding and subtracting positive and negative values. Instructions prompt you along to find your answer.

http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html

http://nlvm.usu.edu/en/nav/frames_asid_162_g_2_t_1.html

NCTM Illuminations Lessons

Illuminations is packed with activities and lessons for grades K–12, many of which can be adapted for adult learners. The lesson "Volt Meter" (http://illuminations.nctm.org/ActivityDetail.aspx?ID=152), gives students an opportunity to see what happens when negative and positive charges come together.

NOVA Online

NOVA has quite a few interesting activities that integrate science, social studies, and math. The activity "Lost at Sea: The Search for Longitude" (http://www.pbs.org/wgbh/nova/teachers/ideas/sammons/) has students developing a latitude formula using positive and negative numbers.

YouTube

YouTube has a multitude of videos. You can do your own search, or check out the videos on signed numbers at
http://www.youtube.com/results?search_query=signed+numbers&search=Search

Integer Addition and Subtraction

This site uses a combination model for illustrating how to add and subtract integers: http://images.google.com/imgres?imgurl=http://www.europa.com/~paulg/mathmodels/VMII-06.gif&imgrefurl=http://www.europa.com/~paulg/mathmodels/int_add_sub.html&h=206&w=366&sz=10&hl=en&start=14&tbnid=ax0FdwXupsXMQM:&tbnh=69&tbnw=122&prev=/images%3Fq=signed

It's All in the Cards: Adding and Subtracting Integers

http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MTMS2007-08-10a. In this article, Gregorio Ponce gives specific directions on how to teach students the game of "It's All in the Cards". He has a website with directions for each level of the game at www.ivcampus.sdsu.edu/math_ed/index_files/itsallinthecards.pps.

Integer Target: Using a Game to Model Integer Addition and Subtraction

In this article by Jerry Burkhart at http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MTMS2007-03-388a, you can learn how to teach your students how to play "Integer Target".

EMPower Materials

The EMPower books that focus on number sense weave frequent opportunities to build number sense throughout the lessons. *Everyday Number Sense: Mental Math and Visual Models* has a good introduction to locating positive and negative numbers on a number line in Chapter 5, *Extending the Line*. http://www.keypress.com/x18177.xml

Key to Algebra

Key to Algebra, Book 1 introduces integer addition and subtraction using a football field for its number line. Key to Algebra Book 1: Operations on Integers. ISBN: 978-1-55953-001-9. https://www.keypress.com/x5205.xml

Ideas

Develop a Human Number Line

Have students create a number line in the hall outside of class or outside. Have them walk forward and backward as they add and subtract integers. Read *Integers Made Easy: Just Walk It Off* by Julie Nurnberger-Haag at http://my.nctm.org/eresources/article_summary.asp?from=B&uri=MTMS2007-09-118a

Building Learning Communities in Multilevel Classes
Problem Statement

One of the principles of the Teachers Investigating Adult Numeracy Project is that students working together and communicating with one another about math increases their opportunities to learn math. This idea is often the one that is the most jarring at first if teachers come from programs with totally individualized math classes. While the motivation behind individualized instruction is understandable—most classes in ABE/GED classes are multileveled, attendance can be erratic, and teachers want to make sure each student gets what is most helpful—we question the effectiveness of silent, lone-wolf math as the primary pedagogical practice.

As we work to transform our classes into the best math learning communities possible, we might consider drawing upon four design characteristics suggested in How People Learn (National Research Council, 2000):

- *the community-centered lens*—developing a culture of questioning, respect, and risk-taking
- *the learner-centered lens*—beginning instruction with prior knowledge and what students think and know
- *the knowledge-centered lens*—being clear about mathematics content to be taught, why it is taught, and evidence of its learning
- *the assessment–centered lens*—providing frequent opportunities for students to make their thinking and leaning visible as a guide for both teachers and students

  (National Research Council, 2005. 12-13 and adapted by Murray & Jorgensen, 2007, 9)

Multilevel classes where students work together on worthwhile mathematical tasks demand skilled facilitation for all students to be included. "Differentiated instruction" is a practice whose goal is to meet the needs of everyone in the class. The authors of *The Differentiated Math Classroom* describe DI this way:

> "First and foremost it is *not* individualized instruction. Differentiated instruction implies a purposeful process for adapting the teaching and learning processes of the classroom to accommodate the needs of all learners. For us, it is an especially useful tool for ensuing that all students have access to and are appropriately supported in their acquisition of important mathematical knowledge. Differentiated instruction encompasses a versatile collection of strategies that have developed over the years, including flexible grouping and tiered activities" (Murray & Jorgensen, 2007).

We invite you to read the account of a Rhode Island TIAN teacher's experience in the classroom working to design a lesson that meets the needs of the students in her multilevel class. You can find her story on the TIAN web site at http://adultnumeracy.terc.edu/TIAN_WS2_RI1.html

Questions to Guide Discussions

After reading the Rhode Island teacher's story, discuss these questions in your local or regional group.

1. In what ways did the teacher pay attention to individuals' needs within the group?

2. Do you have other suggestions or how the teacher could have paid more attention to individual needs but keep the community of learners intact?

3. Use each of the four lenses to describe, evaluate, and suggest next steps for the teacher.

References

Murray M. & Jorgensen, J. (2007). *The differentiated math classroom: a guide for teachers, K-8*. Portsmouth NH: Heinemann.

National Research Council. 2000. *How People Learn: Brain, Mind, Experience, and School*. Expanded Edition. J. Bransford, A. Brown, and R. Cocking, eds. Division of Behavioral and Social Science and Education. Washington, DC: The National Academies Press.

National Research Council. 2005. *How Students Learn: Mathematics in the Classroom.* Committee on *How People Learn,* A Targeted report for Teachers, M.S. Donovan and J. D. Bransford, eds. Division of Behavioral and Social Science and Education. Washington, DC: The National Academies Press.

Introduction to the Facilitator's Guide

Each TIAN Bundle's third section (the Facilitator's Guide) is designed to give some practical suggestions about how to facilitate a teacher meeting using the resources in the other two bundle sections (Math Topic and Teaching/Learning Issue). There is a suggested Meeting Feedback Form for the group and a Teacher Meeting Notes form to send to the tian-talk discussion list by sending an email to tian-talk@cls.coe.utk.edu. Please note these are only suggestions. The TIAN team is interested in hearing what groups decide is most important and helpful for them.

Suggestions for Using Bundle #3 in Teacher Meetings

As you plan to use a Bundle, print out a copy of the entire Bundle (about 40 pages). Read through it, deciding which sections to photocopy for the meeting and which to let group participants access themselves on the TIAN website at http://adultnumeracy.terc.eu/TIAN_teacher_resources.html

If your group has ONE two-hour meeting to spend on Bundle #3, set aside at least 2/3 of the time for the Math Topic and 1/3 of the time on the Teaching/Learning Issue or discussing one or more of the articles from the Articles and References for Teachers. So a meeting might go something like:

1. **Introduce the Math Topic, Integers, or Signed Numbers, either by**
   a. summarizing the main points in the Introduction or
   b. emailing the introduction ahead of time to the group members, and then briefly discuss the main points in the meeting.
2. **Do some math together.**

   There are four activities. Choose two activities to do. We encourage you to begin with 3A Negative Experiences, no matter which activity you follow up with.
3. **Consider the issue: "Multi-Level Classrooms" or discuss the issue of using a particular model after reading an article such as Deborah Loewenberg Ball's "With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics".**

   Ask everyone to read the Problem Statement, then select some of the questions for discussion.

4. **Get some feedback on the meeting and ask a volunteer to send an email to tian-talk to share good ideas that came up in the meeting. Also, ask everyone to bring back to the next meeting what they did with these activities in their classes.**

If your group has TWO two-hour meetings (4 hr) to use Bundle #3, you might spend the entire first meeting on the Math Topic, and the second meeting discussing how things played out in class, ending that second meeting with a discussion of the Teaching/Learning Issue.

In the first meeting, you might have time to do and reflect upon 3 activities, and to begin to chose some articles to read before the next meeting. You might start the second meeting with everyone sharing their feedback based on the article(s) they read.

The Importance of Promoting Teacher Mathematical Learning

ABE math teacher groups get together for two main reasons—to get some good math teaching ideas and resources for their classrooms and to expand their own math knowledge. The activities that you do together begin with teachers wrestling with the problems themselves. As they struggle, some things you do as facilitator will be more likely to promote mathematical learning than others. All facilitators should keep these five important ideas in mind:

1. TIAN teachers value sharing solutions among themselves and encourage sharing in the classroom. When asking people to share, encourage people to explain their thought processes.
2. In the TIAN institutes we were always interested in more than one strategy, and whether we could see the connections between the strategies.
3. Regard confusion and error as learning opportunities—don't avoid it.
4. Raise honest questions that push on the math. This means it is ok to not have the answer to the questions posed. All of us are learners—that includes the facilitator.
5. It's a community—everyone should take responsibility for the learning.

These ideas, so beautifully presented in the table on the next page, would be good for everyone in the group to have a copy of right from the first meeting.

Continuum of Sociomathematical Norms

	Less likely to promote mathematical learning ⟷		More likely to promote mathematical learning
Sharing	Ideas and solutions are shared with minimal or no explanation	Thinking is described, often in procedural terms	Explanations consist of a mathematical argument
Solution Strategies	Emphasis is on one single solution or strategy	Multiple strategies and solutions are described	Emphasis is placed on the relationships among multiple solutions and/or strategies
Confusion & Error	Confusion and mistakes are avoided or ignored, or are corrected by the PD leader	Confusion and mistakes are acknowledged in hopes of causing disequilibrium and change in understanding	Confusion and errors are embraced as opportunities to compare ideas, re-conceptualize problems, explore contradictions in solutions, or pursue alternative strategies
Questioning	The PD leader asks questions aimed at maintaining social order or eliciting specific responses	Both the PD leader and teachers raise procedural and/or factual questions about the mathematics	Both the PD leader and teachers raise questions that push on understanding of mathematics/ mathematical reasoning
Community	Work is generally done individually or ideas are shared through PD leader explication	teachers collaborate to find solutions to problems	Mathematical argumentation forms the basis of a generative learning process where individuals take responsibility for their own and the group's progress

(Adapted from Yackel & Cobb)

Meeting Feedback Form

(for the group and the facilitator)

What was the most effective part of the meeting today, and why?

What would you change for the next time? Why?

What pressing issues/topics would be good to address?

Teacher Meeting Notes

(To share with other groups on the tian-talk discussion list at tian-talk@cls.coe.utk.edu)

Date/time of meeting:

Group Title and meeting location (City or town, State)

Facilitator(s)

Number of participants present

Describe what occurred at the meeting

Did you use any activities or discuss the issue from the TIAN Bundles? How effective were the activities or discussion of the issue?

Did your group use resources others than those in the TIAN Bundles? If so, please describe (or attach).