TEACHERS INVESTIGATING ADULT NUMERACY

Bundle #2

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Math Topic—Operation and Symbol Sense

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Facilitator's Guide

TERC

OPERATION AND SYMBOL SENSE

Introduction

In our work with adult learners—and in our lives—we help others "do math" and do it ourselves. Doing the math is much more than merely cranking out the answer to a calculation. Reasoning and understanding are critical aspects of doing math. This reasoning and understanding is strengthened as students develop operation and symbol sense. "Operation and symbol sense" includes understanding the meanings and models of operations, the real world situations they connect with, and the symbols that represent them. Understanding the operations then is a three-way street that connects visual models, symbols, and situations. In the activities in this Bundle you will connect operations and visual models to consider the four basic operations, odd and even numbers, and the symbol for equality.

A common question in any math class is, "What should I do: add, subtract, multiply, or divide?" We agree with the NCTM statement that learners need to "understand meanings of operations and how they relate to one another" (NCTM, 2000, p. 34). Sometimes revisiting "the basics" can provide us with new insights. This TIAN Bundle intends to provide you with the opportunity to do just that. After all, what could be more basic than the four operations—adding, subtracting, multiplying, and dividing, or the equality symbol?

Visual models are important thinking tools. To communicate mathematically and to flexibly approach problems, adults need visualization and expressive skills. They need to "see" the problem, and they need to know how to express it and their solution processes not only in words and with notation, but with pictures as well. This second TIAN Math Bundle focuses on that idea.

In the math research literature, attention has been drawn to the fact that children do not always have a full understanding of the equal sign as an equality relationship. We wonder if it should it be any different for adults. We ask you to reflect upon your own understanding of this important, in fact, necessary relation that underpins algebraic understanding. The case is made by well respected mathematics education researchers (including Erlwanger and Berlanger (1983); Anenz-Ludlow and Walgamuth (1998); and Carpenter, Franks, and Levi (2003)) who have found that children in elementary grades usually think that the equals sign means to carry out whatever calculation that precedes it and that the number after the equals sign should be the answer to the calculation. They do not see the equals signs as a symbol representing an equivalence relationship.

Researchers such as Knuth, Stephens, McNeil, and Alibali (2006) have also been giving considerable focus to algebraic reasoning, especially because students seem to have an inadequate understanding of and preparation for algebra. Since algebra is considered a gatekeeper for high school students as well as for adult learners, implications from the research likely pertain to all learners. A key finding for the algebra research has focused on the equal sign, supporting the efforts of elementary school researchers. According to the RAND Mathematics Study Panel (2003), "the notion of 'equal' is complex and difficult for students to comprehend" (p. 53). This TIAN Bundle has you reflecting on this critical symbol.

The bundle begins with four activities that ask you as teachers to call upon and expand your own operation and symbol sense, and then to think of ways to integrate activities into your classrooms which purposefully provide opportunities for students to call upon and expand their notions of the operations and equivalence relations. *Activity 2A. The Four Operations: What Do They Mean?* asks you to reflect on your present notion of what the four basic operations mean and consider how to expand those ideas. You are asked to provide visual examples. In *Activity 2B. Cutting Cucumbers*, you are challenged to interpret visual explanations of division problems with fractions. *Activity 2C. Visualizing Odd and Even Results* asks you to develop visual explanations for odd or even results when adding, subtracting, multiplying, and dividing. Finally, *Activity 2D. A Matter of Equality* calls for an exploration of the equal sign symbol as an indicator of an equality relation.

A deep understanding of the operations comes when a person understands various notions for an operation. It is not enough, for example, to conceive of subtraction as taking away one amount from another. The "take-away" model works for having an amount of money, spending some, and figuring out what you have left. However, nothing is taken away when you compare the amount of money you have in the bank with the amount you wish you had. A conception of subtraction as difference or comparison is helpful there. Similarly, division can have two interpretations: either partitive or quotitive.

More classroom resources to develop operation and symbol sense are suggested in *Classroom Resources* and some articles for your personal research and further readings on the topic are listed in *Articles & References*. We also ask that TIAN teachers reflect about how this topic—Operation and Symbol Sense —is reflected in your *State Frameworks*.

Whether you are member of a local or regional group, or working on your own this year, please share additional resources and your thoughts on Math Topic #2 with your TIAN colleagues in Arizona, Kansas, Louisiana, Massachusetts, Ohio, and Rhode Island via the TIAN Talk discussion list. As a TIAN participant, you will automatically be subscribed to the TIAN Talk discussion list. To post a message to this list, send an email to tian-talk@cls.coe.utk.edu. To view the list archives or to manage your email subscription, go to http://cls.coe.utk.edu/mailman/listinfo/tian-talk.

Have Fun!

References

Anenz-Ludlow, Adalira, and Catherine Walgamuth. "Third Graders' Interretations of Equality and the Equal Symbol." *Educational Studies in Mathematics* 35 (1998); 153-87.

Carpenter, Thomaas, Megan Loef Franke, and Linda Levi. *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School*. Heinemann: Portsmouth, NH. 2003.

Erlwanger, Stanley, and Maurice Berlanger. "Interpretations of the Equal Sign among Elementary School Children." In *Proceedings of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Montreal: 1983.

Knuth, Eric, Ana Stephens, Micole McNeile, and Martha Alibali. "Does Understanding the Equal Sign Matter? Evidence from Solving Equations" from *Research in Mathematics Education*, 37 (July 2006): 297-312.

RAND Mathematics Study Panel. (2003). *Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education*. Santa Monica, CA: RAND.

Operation and Symbol Sense

Activity 2A. The Four Operations: What Do They Mean?

Goal: To reflect on our present notions of what the four basic operations mean and consider how to expand those ideas

Time estimate:

30–40 minutes

Focus:

Operation Sense

Materials:

✓ Handout *The Four Operations: What Do They Mean?* (page 6)

Preparation

This activity is designed for teachers first to do together, then take back to their classes to share as is or adapt for their classrooms. PLEASE SHARE INTERESTING POINTS THAT ARISE IN TEACHER or STUDENT DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

5 m. **1.** Ask participants to think about how they remember being taught each of the four basic operations (addition, subtraction, multiplication, and division). How do they explain the meaning of each operation to their students?

8 m. **2.** Distribute the *Operations: What Do They Mean? Handout* and ask participants to complete it as best they can within the time allotted. This part of the activity should not take more than 10 minutes. Notice how individuals are doing. When everyone has at least one definition for each operation jotted down, move on to the next step.

10 m. **3.** Now ask individuals to turn to a partner and take turns sharing their definitions. Be sure to give them time to illustrate some of the

examples, if they want. This may provide some participants an opportunity to build their own understanding of the operations.

10 m. **4.** Bring the entire group back together. Ask

- How did the questions at the bottom of the page influence your thinking about operations?
- Did any of you revise your definition of one or more of the operations? If so, why?

2 m. **5.** Explain that part of the rest of the session will be devoted to answering the questions posed at the bottom of the page.

For facilitators/teachers to think about

Remember to LISTEN and OBSERVE at the beginning.

The purpose of this activity is to unveil people's present understanding of the operations, especially when the operations involve something other than whole numbers, so don't go ahead and teach—"Another way to look at subtraction is ..."

However, if you discover in your class that few people can answer the questions at the bottom of the handout, it will give you ideas for some future lessons.

The Four Operations: What Do They Mean? Handout

Think about when you were taught basic math in school. How were the various operations explained to you? How do those explanations differ (if at all) from how you now explain them to your students?

Jot down how you define each operation. Then think of a way to illustrate your definition in the third column.

Operation	What does it mean?	Show the meaning with chips or something visual
Addition		
Subtraction		
Multiplication		
Division		

Subtraction is more than "Take Away": The idea that subtraction means "take away" matches well with some subtraction scenarios but not all. It's a good match for spending money ("I started with $40, spent $15" I have $25 left). But does "take away" explain the temperature difference between -6 degrees and 6 degrees? Or, if I live 12 miles due west of town and you live 15 miles due west, I live 3 miles from you? What are some other meanings of subtraction?

Are there other meanings for Multiplication?: If you said multiplication is "repeated addition", how do you explain a situation such as 2/3 x 4/5?

Alternate meanings of Division: 12 divided by 6. One person saw two six-packs of Pepsi. Another saw 6 people, each drinking 2 Cokes. Can you visualize it both ways?

Operation and Symbol Sense

Activity 2B. Cutting Cucumbers

Goal: To explore division using visual models

Time estimate:
30–40 minutes

Focus:
Operation Sense

Materials:
- ✓ Handout *Two Ways to "See" Division* (page 10)
- ✓ Handout *Cutting Cucumbers* (page 11)

Preparation
Gather a variety of materials for people to manipulate—pennies, counters, chips, fruit, veggies with knives

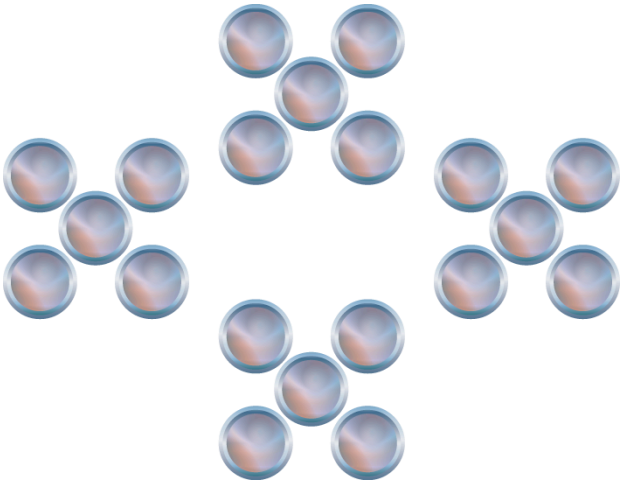
This activity is designed for teachers to do together, and then to think about the implications for their classrooms. PLEASE SHARE INTERESTING COMMENTS THAT ARISE IN TEACHER DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence

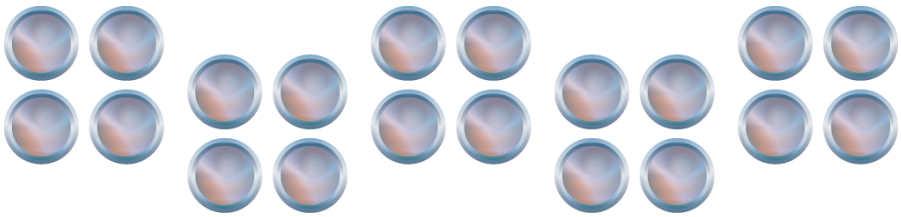
10 m. 1. Write 12÷4 where everyone can see. Ask individuals to show a solution to the division problem with manipulatives.

2. Once everyone has had a chance to do so, ask them to look around to see if all the "pictures" are the same. Most likely. two interpretations of division will turn up. (if not, call attention to the second)

The one that interprets 20 ÷ 5 as "How many 5's are in 20?" THE QUOTITIVE way.



The one that interprets 20 ÷ 5 as "What is 20 shared 5 ways?" or "What is 20 divided into 5 parts?" THE PARTITIVE way.



3. Discuss *Two Division Models Handout*, making sure that everyone understands the difference between the quotitive ("how many in?") and the partitive ("sharing") interpretations.

20 m 1. Explain that at one of the TIAN Institutes, teachers were asked to show with manipulatives these 4 problems (WRITE FOR ALL TO SEE)

1. 12 ÷ 4
2. ½ ÷ 4
3. ½ ÷¼
4. 2/3 ÷ ¼

2. Then explain that during that Institute, one group got carried away with the salad left over from lunch and displayed their own division

challenge (what you see in the "Cucumber Division" photo). Can you match the symbolic division with its cucumber model?

3. Once pairs have a chance to complete the Cucumber Division matching, ask them to consider:
   - What were they thinking as they matched pictures to models?
   - Does one interpretation—" The how many in?" or the "sharing"—make more sense in the particular cases?

10 m. 1. Bring everyone together. First discuss the interpretations of the models.
Be sure to decide who is going to share the interpretations on the TIAN –talk listserve.

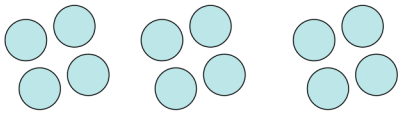
2. Then, focus on the classroom
   - What are the implications for teaching?
   - What happens if you only teach one interpretation of division?

Two Ways to "See" Division Handout

Different ways to "see" division

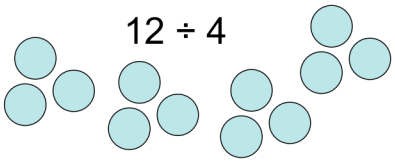
- Quotitive
  - Measurement model
  - "How many 4's in 12?"

12 ÷ 4



- Partitive
  - Sharing
  - Splitting
  - "How many in each of the 4 groups?
  - Breaking it up evenly

12 ÷ 4



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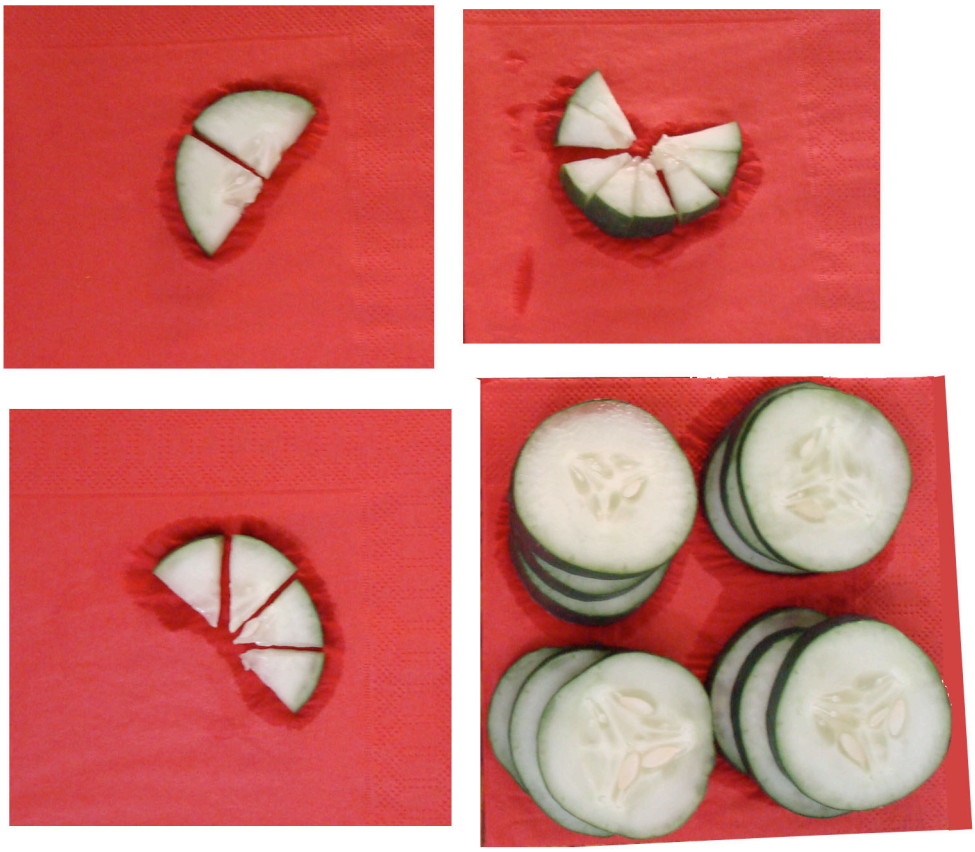
Cucumber Division Handout



Operation and Symbol Sense

Activity 2C. Visualizing Odd and Even Results

Goal: To develop visual explanations for odd or even results when adding, subtracting, multiplying, and dividing

Time estimate:

30–40 minutes

Focus:

Operation Sense

Materials:

- ✓ Handout *Odd and Even* (pages 15–16)
- ✓ Some chips, paper clips, or pennies to manipulate

Preparation

For visualization examples, read Visualizing the Numbers (Part II) http://www.suite101.com/article.cfm/math_fun/73280 and Visualizing the Numbers (Part IV) http://www.suite101.com/article.cfm/math_fun/75894 .

This activity is designed for teachers first to do together, and then to bring back to their classes to share as is or adapt for their classrooms. PLEASE SHARE INTERESTING VISUALIZATIONS THAT ARISE IN TEACHER or STUDENT DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

2 m. **1.** There are some interesting patterns that happen with operations with odd and even numbers. This activity gives participants a chance to explore visually what those patterns look like.

15 m. **2.** Distribute the *Odd and Even Handout* and ask participants to complete it as best they can within the time allotted. Before they start, throw out a handful of pennies or chips on the table, and ask, "How can you tell if this amount is even or odd without actually counting it?" (One response is that you can pair objects off, and have nothing left over.)

This part of the activity may take a while since most have probably not tried to visualize such rules before. Notice how individuals are doing. When everyone has at least one or two examples jotted down, or when you sense that participants are getting frustrated, move on to the next step.

5 m. **3.** Now ask individuals to turn to a partner and take turns sharing their examples and conclusions. Do they agree?

10 m. **4.** Bring the entire group back together.
First, discuss the math.
Ask:

- Were there any disagreements?
- Any surprises or discoveries?
- How would this help you know that 8 x 91 could never be 71? Or, 73 + 234 could never be 306?

Next, discuss what might happen in the ABE classroom:

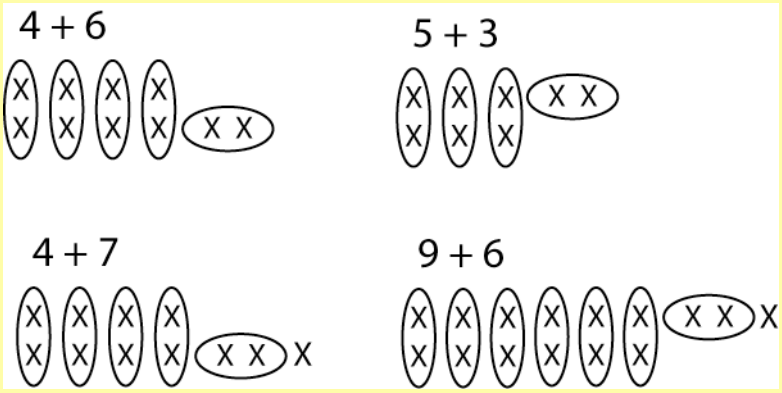
- What was challenging about this activity?
- How useful do you think it is to have students create visual models in the classroom?
- How might you adapt this activity for your class?

For facilitators/teachers to think about

Remember to LISTEN and OBSERVE at the beginning.

The purpose of this activity is to have people develop their own visualization strategies, so don't go ahead and teach—"Here's how you draw ..."

However, if you discover in your class that few people seem to be able to get started, you might explore addition together, using a variety of pairs for even plus even, odd plus odd, even plus even, odd plus even. Make sure to push for the visual representation, for example:



Odd and Even Handout

Work with a partner to draw pictures of objects to show an example of the odd/even rules for addition and subtraction on the first page and for multiplication and division on the second page. You may want to play around with objects like pennies or chips before you draw.

	Addition	Sum (even or odd?)	Subtraction	Difference (even or odd?)
Even and even				
Even and odd				
Odd and even				
Odd and odd				

	Multiplication	Product (even or odd?)	Division	Quotient (even or odd?)
Even and even				
Even and odd				
Odd and even				
Odd and odd				

Operation and Symbol Sense

Activity 2D. A Matter of Equality

Goal: To explore the importance of understanding the equal sign symbol as an indicator of an equality relation.

Time estimate:

45 minutes

Focus:

Symbol Sense—the Equal Sign

Materials:

✓ Handout *A Matter of Equality* (page 19)

Optional: hard copies of the research articles—also in softcopy, available online

Preparation

Read Children's Understanding of Equality: A Foundation for Algebra: *http://www.wcer.wisc.edu/ncisla/publications/articles/AlgebraNCTM.pdf*

This activity is designed for teachers to do together, to discuss the results, and to question their own understanding and pedagogy. PLEASE SHARE INTERESTING COMMENTS OR QUESTIONS THAT ARISE IN TEACHER or STUDENT DISCUSSIONS ON THE "TIAN Talk" DISCUSSION LIST at tian-talk@cls.coe.utk.edu.

Suggested Activity Sequence:

10 m. **1.** Distribute the handout and ask participants to individually respond to the questions as best they can.

20 m. **2.** Now go over the responses with participants. First have volunteers share their responses, then share some of the research findings associated with each question:

Question 1. Rather than thinking about the equality relation, students typically think of the equal sign as an indication to "do it"—whatever the operation is preceding the equal sign. Researchers in one study

were able to correlate students' responses to "what does the symbol mean" to their ability to algebraically solve simple problems such as 4m + 10 = 70. [Knuth and Stephens. Does Understanding the Equal Sign Matter? Evidence from Solving Equations]

Question 2. Students who were asked this question typically responded with 12 because they believed the = sign meant to perform the operation coming before it. [See the results of students from 1st–6th grade in Falkner et al's article *Children's Understanding of Equality: A Foundation for Algebra*.]

Question 3. Students who see the = sign as a relationship between the two sides can readily figure out the missing number without having to add the numbers on the left and then subtract the number on the right. Instead, they see the relationship between 86 and 84 and can therefore figure out what number would relate to 57. [from Carpenter, Franke, Levi's book *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*.]

Question 4. This is an example of the type of activity teachers can be using with their students in order to practice looking for relationships on both sides of the = sign. [from Carpenter, Franke, Levi's book *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*, p. 41]

5 m. **3.** Have participants get into groups of 2–4 and discuss these questions: *If this is what research is saying about why some younger students have trouble with algebra, do you think this might also be true of some of our adult learners? If so, what should we do?*

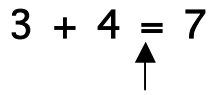
10 m. **4.** Ask each group to share a teaching strategy and capture on chart paper so participants can write them down and develop their own bank of strategies. Share these strategies on the "tian talk" discussion list.

For facilitators/teachers to think about

Remember to LISTEN and OBSERVE and respect participants' responses, especially if they are contrary to what the research suggests.

A Matter of Equality Handout

1. The following questions are about this statement:



   a) The arrow above points to a symbol. What is the name of the symbol?

   b) When you enter these symbols into a calculator, the "=" usually means "OK here comes the answer", but what are other ways to think about what "=" means?

2. What number should go in the ☐ for this statement: 8 + 4 = ☐ + 5 ?
   What numbers do you think your students might give as answers?

   Why?

3. What number should go in the ☐ for this statement: 57 + 86 = ☐ + 84 ?

   How did you figure out the missing number?

4. For each statement below, try to decide whether each statement is true by looking at the equality relations. Put a check beside each true statement.

   a) _____ 33 – 27 = 34 – 26

   b) _____ 583 – 529 = 83 – 29

   c) _____ 5 x 84 = 10 x 42

   d) _____ 64 + 14 = 32 + 28

   e) _____ 60 x 48 = 6 x 480

   f) _____ 471 – 382 = 474 - 385

Operation and Symbol Sense

Connecting to State Standards

How and where (at which levels and under which math topics) do your state math standards or benchmarks address the topic of operation sense? Are multiple ways of understanding the operations, especially with visual models explicitly described? Implicitly assumed?

When is understanding of symbols such as the equal sign and the concept of equality addressed? Do you think your state standards do a good job of anticipating algebra in the arithmetic section?

Below are links to each of the 6 states standards web pages.

Arizona

http://www.ade.az.gov/adult-ed/adult_ed_standards.asp

Kansas

http://adultnumeracy.terc.edu/pdfs/KS_state_standards.pdf

Louisiana

http://www.doa.louisiana.gov/osr/lac/28v129/28v129.doc.

Massachusetts

http://www.doe.mass.edu/acls/frameworks/

Ohio

http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&TopicRelationID=966&Content=21875

Rhode Island

http://www.brown.edu/lrri/standards.html,

Articles and References (For Teachers) About Strengthening Operation and Symbol Sense

Read and discuss some of these articles with fellow teachers. Use the attached Research Reading Response (p. 23) to help guide your discussions. Please share your thoughts with others by posting online to the TIAN Talk discussion list at tian-talk@cls.coe.utk.edu.

About Operation Sense

Gregg, Jeff and Diana Gregg. "Measurement and Fair-Sharing Models for Dividing Fractions" from *Mathematics Teaching in the Middle School*, 12 (May 2007): 490–496. Includes examples of how to move students from division of whole numbers to division of fractions.

Lamberg, Teruni de Silva. "Student Approaches to Unitizing in Fair-Share Problems" from *Mathematics Teaching in the Middle School*, 13 (September 2007): 114–116. This short article analyzes four students' methods for dividing (fair-share situation).

Nugent, Patricia. "Lattice Multiplication in a Preservice Classroom" from *Mathematics Teaching in the Middle School*, 13 (September 2007): 110–113. Lattice multiplication is an alternative algorithm for teaching multiplication that helps students see the place values.

Taber, Susan. "Using Alice in Wonderland to Teach Multiplication of Fractions" from from *Mathematics Teaching in the Middle School*, 12 (December 2006): 244–249. Although practitioners won't be reading Alice in Wonderland to her students, this article does provide examples of the difference between proportional and nonproportional thinking with fractions.

Tent. Margaret. "Understanding the Properties of Arithmetic: A Prerequisite of Arithmetic" from *Mathematics Teaching in the Middle School*, 12 (August 2006): 22–25. Provides strategies for teaching the commutative, associative, distributive, and identity properties of addition and multiplication.

About the Understanding of the Equal Sign

Falkner, Karen, Linda Levi and Thomas Carpenter. Children's Understanding of Equality: A Foundation for Algebra. http://www.wcer.wisc.edu/ncisla/publications/articles/AlgebraNCTM.pdf. This brief article describes the misconceptions students develop at a very early age about the equal sign. Included are strategies one teacher used to try to overcome those misconceptions.

Freiman, Viktor and Lesley Lee. "Tracking Primary Students' Understanding of the Equality Sign". Proceedings of the 28th Conference of the International Group for the Psychology of

Mathematics Education, 2004 (vol.2, pp. 415 – 422). http://www.emis.de/proceedings/PME28/RR/RR236_Freiman.pdf. This research was aimed at tracking young students' understanding of the equal sign by monitoring how they responded to simple problem types such as $a + b = c$ and $c = a + b$. Types of errors in children in K, grade 3, and grade 6 were then analyzed.

Knuth, Eric and Ana Stephens. "Does Understanding the Equal Sign Matter? Evidence from Solving Equations" from Research in Mathematics Education, 37 (July 2006): 297–312. This research project investigated the relationship between being able to solve simple algebraic equations and an understanding of the equal sign.

Other Related Articles

Algebraic Thinking from Assessment Resource Banks: English, Mathematics, and Science. http://arb.nzcer.org.nz/supportmaterials/Maths/concept_map_algebraic.php This collection of mini-resources includes hands-on strategies for teaching equality and the additive, commutative, and associative properties, beginning very early on in learners' math education.

Computation, Calculators, and Common Sense (May 2005). (a position paper from the National Council of Teachers of Mathematics) http://www.nctm.org/about/content.aspx?id=6358. A reaction to the question about calculator use in the classroom.

National Research Council (2001). Number: What Is There to Know?" from *Adding It Up: Helping Children Learn* (71–114).

Schifter, Deborah, Virginia Bastable, Susan Jo Russell, Lisa Seyferth, and Margaret Riddle (2006). Algebra in the K–5 Classroom: Learning Opportunities for Students and Teachers. http://www2.edc.org/CDT/docs/Papers/AlgNCTMYrbk.pdf. Discussion of classroom investigations into how students develop algebraic thinking.

Stiff, Lee (2001). Making Calculator Use Add Up. http://www.nctm.org/about/content.aspx?id=1242. A response to a newspaper article about the use of calculators in the classroom.

Research Reading Response

Title/Author/Reference:

_____

_____

Main Ideas:

- What were the author's main ideas regarding adults/children *learning* mathematics?

- What were the author's main ideas regarding *teaching* mathematics to adults/children?

Applications:

- What are the implications for you as an ABE mathematics teacher?

Classroom Resources
To Strengthen Operation and Symbol Sense

This is a starter list of classroom resources that focus on strengthening operation sense and those symbols associated with the operations, including the equal sign. If you know of other resources, please share with others by posting online to the TIAN Talk Discussion list at tian-talk@cls.coe.utk.edu.

Published and Online Resources

EMPower Materials

The EMPower books that focus on number sense weave frequent opportunities to build number sense throughout the lessons. Below are some locations where operation sense are developed.

Everyday Number Sense: Mental Math and Visual Models includes opportunities for students to build their operation sense. Students use visual models and real-life situations to explore the basic operations.

Operation Sense: Even More Fractions, Decimals, and Percents focuses entirely on operation sense, but with fractions, decimals, and percents. Students actually "see" why a fraction divided by a fraction sometimes yields a larger number. Lesson 6, *Division: How Many ___ in ___?*, explores the two meanings of a division equation and then focuses on how to model division with fractions and is available free online at http://www.keypress.com/x18176.xml.

NCTM Illuminations Lessons

Illuminations is packed with activities and lessons for grades K–12, many of which can be adapted for adult learners. The lesson "Factorize" (http://illuminations.nctm.org/ActivityDetail.aspx?ID=64), gives students an opportunity to visually explore factors of numbers between 1 and 49. Another activity, "Pan Balance—Numbers", (http://illuminations.nctm.org/ActivityDetail.aspx?ID=26) provides students with interactive experiences to develop the understanding that the equal sign signifies a relationship, not an operation.

National Library of Virtual Manipulatives

The National Library of Virtual Manipulatives from the University of Utah (http://nlvm.usu.edu/en/nav/topic_t_1.html) offers several interactive activities for students to work on developing operation sense. For example, Diffy (http://nlvm.usu.edu/en/nav/frames_asid_326_g_4_t_1.html) is designed to just reinforce subtraction sense, using whole numbers, fractions, and signed numbers. Number Line Bounce (http://nlvm.usu.edu/en/nav/frames_asid_107_g_1_t_1.html) allows students to visually represent a sequence of operation on a number line to create number sentences.

Problems with a Point

A resource of downloadable activities at a variety of levels, available at http://www2.edc.org/mathproblems/searchTopic.asp. For example, to further explore the idea of even/odd, you might want to ask your students to work on the challenging problems related to the idea of even/odd http://www2.edc.org/mathproblems/problems/printProblems/nsParity.pdf). Or, have students try the Adding Digits Problem (http://www2.edc.org/mathproblems/problems/printProblems/bkAddingDigits.pdf) where they begin to articulate some basic principles related to operation sense

Ideas

Who Am I?

This activity focuses on fluency with the operations. Orally present questions such as "I am an odd number between 60 and 65 and my two digits add up to 9. Who am I?" to challenge students to think critically about number relationships. As students become more proficient in this activity, try different operations with more challenging questions, such as "My first and second digits are the same odd number. I am a multiple of 3 that is greater than 50 but less than 100. Who am I?" An alternative to asking questions orally is to have students pair off and challenge each other, making them develop their own questions.

Make 12

This activity helps students think fluently about number operations. Hand out four index cards with numbers, such as 2, 4, 6, and 8, written on each. Students have to use the four numbers and any or all operations to try to reach the target number—in this case 12 (for example: 8 + 6 – 4 + 2 = 12). You can vary the activity by changing the target, for example, "Make 20". Depending on the level of your students, you can give them index cards with single-digit, double-digit, or even benchmark fractions.

Start Here…End Here

(Petreshene, 1985) This activity has students using various operations to get from the first number to the last. For example, if the numbers 2 and 4 were given, students would have to use any combination of operations and other numbers to get from 2 to 4. Sample answers could be: 2 x 7 – 10 = 4; 2 ÷ 2 x 5 – 1 = 4. Encourage students to try to come up with at least three combinations. Have them partner with another student so that they can check each other's work.

Algorithms* or Models?
Problem Statement

Scenario 1

Many students in Joan's ABE math class have been working for a month on adding fractions and mixed numbers with like and unlike denominators. They have been practicing with several pages of computation out of a workbook, and seem to be getting better, even though many still use their times tables as an aid. With all this practice and opportunity to work on fractions, Joan doesn't understand why no one is able to easily solve the problem, "What is 5 ½ + 12 3/4 ?"

Scenario 2

In her ABE math class, Abby provides her students with plenty of practical and concrete opportunities to reason with common "benchmark" fractions such as halves, quarters, eighths, and tenths. They develop and share a variety of strategies. They can confidently solve problems like "What is 5 ½ + 12 3/4 ?" While most are very good at estimating answers to problems like "5 5/12 + 8 19/24" (a little more than 14), they have not spent much time learning the method taught in the workbooks, so they tend to reach for the calculator in that case.

Among math teachers at all levels, there has been much discussion about teaching for understanding vs. teaching algorithms. In adult education, we have an added dilemma because usually our students are with us for a short period of time. We have to make the most of their time while they are with us. So, do we show them the most "efficient" way to solve a problem, or do we try to ensure that they have an understanding of the concept, taking time to ask them to visualize or create models of critical concepts in math? When do we just teach the process (or algorithm)? When do we ask students to develop their own models for understanding?

Consider the questions on the next page with your regional or local group:

* An algorithm is a step-by-step procedure designed to achieve a certain objective in a finite time for efficiency's sake, often with several steps that repeat or "loop" as many times as necessary. The most familiar algorithms are the procedures for the four basic operations: adding, subtracting, multiplying, and dividing, but there are many other algorithms in mathematics.

Questions to Guide Discussions

Discuss these questions in local or regional groups

1. What advice would you give the teachers (Joan and Abby) in each of the scenarios?

2. When do *you* teach algorithms? When do *you* teach for understanding?

3. How do you reconcile the difference between wanting to teach for understanding and showing students the most efficient generalized method?

4. From your experience, how would you describe your students' ability to use math when they enter your classroom? When they leave your classroom after a period of instruction?

5. According to Brophy (Brophy, Jere. "Probing the Subtleties of Subject-Matter Teaching." EDUCATIONAL LEADERSHIP (April 1992): 4-8), *Current research, while building on findings indicating the vital role teachers play in stimulating student learning, also focuses on the role of the student. It recognizes that students do not merely passively receive or copy input from teachers, but instead actively mediate it by trying to make sense of it and to relate it to what they already know (or think they know) about the topic. Thus, students develop new knowledge through a process of active construction. In order to get beyond rote memorization to achieve true understanding, they need to develop and integrate a network of associations linking new input to preexisting knowledge and beliefs anchored in concrete experience. Thus, teaching involves inducing conceptual change in students, not infusing knowledge into a vacuum* (p.5). Do you agree or disagree with his comments? Why? How might your beliefs influence your teaching?

Suggested advance readings

"Teaching for Understanding: Educating Students for Performance" http://www.weac.org/resource/june96/under.htm for a review on beliefs about learning and about teaching

"Concept vs. Computation: The Teacher's Role" http://www.math.uic.edu/~jbaldwin/pub/mamer.html reviews some of the findings from Dr. Ma's significant study on how American teachers teach basic operations.

Introduction to the Facilitator's Guide

Each TIAN Bundle's third section (the Facilitator's Guide) is designed to give some practical suggestions about how to facilitate a teacher meeting using the resources in the other two bundle sections (Math Topic and Teaching/Learning Issue). There is a suggested Meeting Feedback Form for the group and a Teacher Meeting Notes form to send to the tian-talk discussion list by sending an email to tian-talk@cls.coe.utk.edu. Please note these are only suggestions. The TIAN team is interested in hearing what groups decide is most important and helpful for them.

Suggestions for Using Bundle #2 in Teacher Meetings

As you plan to use a Bundle, print out a copy of the entire Bundle (about 40 pages). Read through it, deciding which sections to photocopy for the meeting and which to let group participants access themselves on the TIAN website at http://adultnumeracy.terc.eu/TIAN_teacher_resources.html

If your group has ONE two-hour meeting to spend on Bundle #2, set aside at least 2/3 of the time for the Math Topic and 1/3 of the time on the Teaching/Learning Issue. So a meeting might go something like:

1. **Introduce the Math Topic, Operation and Symbol Sense, either by**
   a. summarizing the main points in the Introduction or
   b. emailing the introduction ahead of time to the group members, and then briefly discuss the main points in the meeting.
2. **Do some math together.**

   There are four activities. Choose two activities to do, for example, 2A the Four Operations: What Do They Mean? and 2D A Matter of Equality.
3. **Consider the issue: "Algorithms or Models?"**

   Ask everyone to read the Problem Statement, then select some of the questions for discussion.

4. **Get some feedback on the meeting and ask a volunteer to send an email to tian-talk to share good ideas that came up in the meeting. Also, ask everyone to bring back to the next meeting what they did with these activities in their classes.**

If your group has TWO two-hour meetings (4 hr) to use Bundle #2, you might spend the entire first meeting on the Math Topic, and the second meeting discussing how things played out in class, ending that second meeting with a discussion of the Teaching/Learning Issue.

In the first meeting, you might have time to do and reflect upon 3 activities, and to begin to chose some articles to read before the next meeting. You might start the second meeting with everyone sharing their feedback based on the article(s) they read.

The Importance of Promoting Teacher Mathematical Learning

ABE math teacher groups get together for two main reasons—to get some good math teaching ideas and resources for their classrooms and to expand their own math knowledge. The activities that you do together begin with teachers wrestling with the problems themselves. As they struggle, some things you do as facilitator will be more likely to promote mathematical learning than others. All facilitators should keep these five important ideas in mind:

1. TIAN teachers value sharing solutions among themselves and encourage sharing in the classroom. When asking people to share, encourage people to explain their thought processes.
2. In the TIAN institutes we were always interested in more than one strategy, and whether we could see the connections between the strategies.
3. Regard confusion and error as learning opportunities—don't avoid it.
4. Raise honest questions that push on the math. This means it is ok to not have the answer to the questions posed. All of us are learners—that includes the facilitator.
5. It's a community—everyone should take responsibility for the learning.

These ideas, so beautifully presented in the table on the next page, would be good for everyone in the group to have a copy of right from the first meeting.

Continuum of Sociomathematical Norms

	Less likely to promote mathematical learning ⟷		More likely to promote mathematical learning
Sharing	Ideas and solutions are shared with minimal or no explanation	Thinking is described, often in procedural terms	Explanations consist of a mathematical argument
Solution Strategies	Emphasis is on one single solution or strategy	Multiple strategies and solutions are described	Emphasis is placed on the relationships among multiple solutions and/or strategies
Confusion & Error	Confusion and mistakes are avoided or ignored, or are corrected by the PD leader	Confusion and mistakes are acknowledged in hopes of causing disequilibrium and change in understanding	Confusion and errors are embraced as opportunities to compare ideas, re-conceptualize problems, explore contradictions in solutions, or pursue alternative strategies
Questioning	The PD leader asks questions aimed at maintaining social order or eliciting specific responses	Both the PD leader and teachers raise procedural and/or factual questions about the mathematics	Both the PD leader and teachers raise questions that push on understanding of mathematics/ mathematical reasoning
Community	Work is generally done individually or ideas are shared through PD leader explication	teachers collaborate to find solutions to problems	Mathematical argumentation forms the basis of a generative learning process where individuals take responsibility for their own and the group's progress

(Adapted from Yackel & Cobb)

Meeting Feedback Form

(for the group and the facilitator)

What was the most effective part of the meeting today, and why?

What would you change for the next time? Why?

What pressing issues/topics would be good to address?

Teacher Meeting Notes

(To share with other groups on the tian-talk discussion list at tian-talk@cls.coe.utk.edu)

Date/time of meeting:

Group Title and meeting location (City or town, State)

Facilitator(s)

Number of participants present

Describe what occurred at the meeting

Did you use any activities or discuss the issue from the TIAN Bundles? How effective were the activities or discussion of the issue?

Did your group use resources others than those in the TIAN Bundles? If so, please describe (or attach).