REVEAL

Instrument/Measure: *Mathematical Reasoning at Exhibits Coding Rubric*

Type:	Video coding rubric
Number of items:	Four coding dimensions per rubric
Primary construct:	Level of mathematical reasoning by families at exhibits
Intended audience:	Intergenerational visitor groups in a museum
Language(s):	English, Spanish
Suggested citation:	Pattison, S., Rubin, A., Benne, M., Gontan, I., Andanen, E., Shagott, T., Francisco, M., Ramos-Montañez, S., Bromley, C., & Dierking, L. (2016). *The impact of facilitation by museum educators on family learning at interactive exhibits: Results of a quasi-experimental study*. Manuscript in preparation.

The Mathematical Reasoning at Exhibits Coding Rubric (MRE) is designed to measure the level of mathematical reasoning expressed by families and intergenerational visitor groups engaging with interactive math exhibits. The MRE focuses on mathematical reasoning related to algebraic thinking and functions (Kaput et al., 2008) and assesses verbal and behavioral indicators of reasoning by group members across four dimensions: (a) talking about mathematical quantities, (b) describing mathematical relationships among those quantities, (c) exploring mathematical relationships in the exhibit, and (d) achieving mathematical goals. For each dimension, coders watch videotaped visitor interactions and rate the level of mathematical reasoning from one (no indicators present) to five (highest level of indicators present). As part of the REVEAL project (https://REVEAL.terc.edu), three versions of the MRE were developed specific to three different exhibits included in the study. MRE scores for each visitor group are constructed using the unweighted average of all four dimensions.

Development process

The full MRE development process is described in Pattison et al. (2016) and included initial conceptualization of the four dimensions of mathematical reasoning (Pattison, Randol, et al., 2016), operationalization of the dimensions and piloting by the research team, formal testing and interrater reliability assessment with four new coders, testing by two bilingual/bicultural researchers with video of Spanish-speaking visitors, and final reliability and validity assessment with 263 family groups.

Cultural assumptions and considerations

- The MRE only assesses verbal and behavioral indicators of mathematical reasoning and does not measure unexpressed mental reasoning. The rubric assesses group-level reasoning, at the highest level demonstrated by any group member.
- The MRE was intended to balance verbal indicators of mathematical reasoning (e.g., talk about mathematical quantities) with behavior indicators (e.g., testing mathematical relationships using the exhibits). However, the final rubric relies more on verbal than non-verbal indicators.
- Because the MRE focuses on mathematical reasoning related to algebraic thinking, it represents only one aspect of the mathematics that visitors might engage with at an exhibit.
- There were no statistically significant correlations between the MRE and participant gender, race, languages spoken at home, or education level. However, participants that reported speaking a language other than English at home scored significantly lower, on average, compared to those that that reported speaking only English.

Reliability and validity evidence

- Interrater reliability for the MRE was high. Intraclass correlation coefficients ranged from 0.79 to 0.84 for the final video coding, indicating that the majority of variation across ratings (at least 79%) was attributable to differences among participant groups, rather than among raters.
- Internal consistency, or reliability across dimensions, was also strong for each exhibit (Cronbach's α from 0.67 to 0.82).

References

Kaput, J. J., Carraher, D. W., & Blanton, M. L. (2008). *Algebra in the early grades*. New York: Lawrence Erlbaum Associates/National Council of Teachers of Mathematics.

Pattison, S. A., Randol, S., Benne, M., Rubin, A., Gontan, I., Andanen, E., … Dierking, L. D. (2016). *Modeling staff-facilitated family learning at interactive math exhibits: A design- based research study*. Manuscript in preparation.

Pattison, S. A., Rubin, A., Benne, M., Gontan, I., Andanen, E., Shagott, T., … Dierking, L. D. (2016). *The impact of facilitation by museum educators on family learning at interactive exhibits: Results of a quasi-experimental study*. Manuscript in preparation.

Mathematical Reasoning Rubrics

Math Reasoning Behavior Checklist: *Balancing Art*

Coder initials: _______ Date:_______ Group #:_______

	Level 2	Level 3	Level 4	Level 5	Rating
Talking about mathematical quantities	☐ Verbalizes number labels on weights ☐ Verbalizes number labels on bar		☐ Mentions equals or equivalence (not counting "balance") ☐ Mentions distance from center or farther/closer ☐ Mentions heaviness, weight, or force		**(See below)** **Level 1: No boxes checked** Rating:______
Describing mathematical relationships	☐ States that both weight and distance matter relative to force, balance, or "heaviness"	☐ States that the farther out, the heavier ☐ States that some combination of distance and weight on both sides have to be equal	☐ Describes a specific *quantitative* case, with numbers, an operator, and an equal sign (e.g., 2 x 3 = 6) ☐ States that weight needs to be multiplied by distance	☐ States that the sum of weight times distance must be equal on both sides for bar to balance	**Highest level checked** **Level 1: No boxes checked** Rating:______
Exploring mathematical relationships	☐ Places, replaces, or moves weight *incorrectly* after checking balance	☐ Moves, replaces, or adds weight *correctly* after checking balance	☐ Verbalizes or writes calculation and then places weight (no clear prediction verbalized or written)	☐ Verbalizes or writes calculation, verbalizes or writes prediction of needed weight and location, and then places weight	**Highest level checked** **Level 1: No boxes checked** Rating:______
Achieving mathematical goals	☐ Balances symmetric configurations Number: __________ ☐ Balances additive symmetric configurations Number: __________ ☐ Balances inverse configurations Number: __________		☐ Balances asymmetric configurations Number: __________		**(See below)** **Level 1: No boxes checked** Rating:______

Rating math quantities for Balancing Art

- Level 1: No boxes checked.
- Levels 2-3: Level 2 for verbalizing either weight or distance labels, level 3 for both.
- Levels 4-5: Level 4 for verbalizing both weight and distance labels AND mentioning one or two types of quantity indicator words (i.e., one or two boxes checked). Level 5 for verbalizing both weight and distance labels AND mentioning all three types of quantity indicator words (i.e., three boxes checked). *Level 3 for mentioning one or more quantity indicator words but not verbalizing both weight and distance labels.*

Rating mathematical goals for Balancing Art

- Level 1: No boxes checked.
- Level 2: One symmetric configuration, no other configurations.
- Level 3: More than one symmetric configurations OR one or more additive symmetric configurations OR one or more inverse configurations (i.e., anything beyond one symmetric configuration but WITHOUT any asymmetric configurations).
- Levels 4-5: *Level 4 for one asymmetric configuration, level 5 for more than one. Levels 2 and 3 do not need to be achieved to be rated at levels 4 or 5.*

General mathematical reasoning coding instructions

- Families do not have to use the exact language stated in the rubric but can be coded for phrases with equivalent meaning.
- All family member talk and behaviors, from both children and adults, are included in ratings. Behaviors and talk can come from any family member and do not need to be restated or even acknowledged by the rest of the group.
- Facilitator talk and behaviors are not included in ratings. However, if visitors contribute substantively to a phrase or question-answer sequence that is initiated by the facilitator, the whole phrase or sequence can be coded. For example, the facilitator might begin a sentence, “the farther the weight is from the center…,” and the visitor might finish, “the heavier it is.” In this case, the whole phrase would count towards “level 3” describing mathematical relationships.
- Do not rate interactions based on your perceptions of visitors’ understanding of the exhibits or the mathematics. Apply the checklist and ratings literally, as described in the rubric.
- Visitor talk is rated the same whether it is in the form of a question or a statement.
- For describing mathematical relationships, quantities must be connected grammatically by visitors (or by a combination of staff and visitor comments), rather than simply stated separately.

Balancing Art-specific coding instructions

- For levels 4 and 5 of talking about mathematical quantities, the Spanish verb “balancear” is considered equivalent to “balance” but the verb “equilibrar” counts as a mention of equals or equivalence.
- Incorrectly using only addition to describe relationships between two sides does not count as a quantitative case, relationship, or verbalized or written calculation.
- For level 3 of describing mathematical relationships, “correct” means in the appropriate direction, in terms of weight or distance, to achieve balance based on the current configuration (e.g., adding more weight to one side that is currently “lighter” than the other).
- For level 4 of describing mathematical relationships, specific quantitative cases must clearly be in reference to weights and distances on the beam, rather than to an unrelated math problem.
- For mathematical exploration, if visitors appear like they might be doing mental math but do not write or verbalize any calculations, they should be coded as level 3.
- For achieving mathematical goals, groups do not need to balance a symmetric or inverse configuration to be counted at levels 4 or 5. Balancing just one asymmetric configuration counts as level 4. Configurations with mystery weights always count as asymmetric.
- For achieving mathematical goals, symmetry is based on piece weight, not piece shape. A configuration that has the same weight pieces on each side is symmetric, even if the shapes of the pieces are different.
- (See table below for definitions of different types of balanced configurations.)

Configuration type	Definition	Example
Symmetric	Same weights at the same distances on both sides. Symmetry is based on piece weight, not piece shape. A configuration that has the same weight pieces on each side is symmetric, even if the shapes are different.	(4 at 2) <> (4 at 2) (3 at 2) and (2 at 5) <> (3 at 2) and (2 at 5)
Additive symmetric	Same weights at the same distances on both sides except that on one side, a single “weight” is made up of multiple weights hung together. More complicated additive symmetric patterns involving weights hung on more than one hole on each side are counted as asymmetric.	((1+1) at 3) <> (2 at 3) ((4 at 3) <> ((2+2) at 3)
Inverse	A single weight and distance pairing on one side matched with the reversed weight-distance pairing on the other side. More complicated inverse patterns involving more than one weight on each side are counted as asymmetric.	(4 at 3) <> (3 at 4) (2 at 4) <> (4 at 2)
Asymmetric	Any configuration that does not count as symmetric, additive symmetric, or inverse.	(3 at 2) and (1 at 5) <> (3 at 1) and (4 at 2) ((1+2) at 2) and (1 at 5) <> (3 at 2) and (1 at 5) (4 at 3) and (2 at 2) <> (3 at 4) and (2 at 2)

Math Reasoning Behavior Checklist: *Drawing in Motion*

Coder initials:_______ Date:_______ Group #:_______

	Level 2	Level 3	Level 4	Level 5	Rating
Talking about mathematical quantities	☐ Verbalizes number (and possibly direction on the slider) associated with the x-axis (e.g. "you go to 4," "go up to 9," "you should be at 5") ☐ Verbalizes number (and possibly direction on the slider) associated with the y-axis (e.g. "you go down to 2," "move to 10 now," "I stay at 5 and you move to 6")		☐ Describes direction and/or shape of line on screen, using words such as: "vertical," "horizontal," "up-and-down," "back-and-forth," "at an angle," or "diagonal" Note each occurrence: ______________________________ ☐ Uses math language to describe point location or slider motion, e.g. "I'm X and you're Y," "we move to (3,4)" Note each occurrence: ______________________________		**(See below)** **Level 1: No boxes checked** Rating:______
Describing mathematical relationships	☐ States that people have to move together to make a diagonal line but not that their relative speeds matter (does NOT need to use the word "diagonal")	☐ Mentions a qualitative relationship between line and relative speeds of sliders (e.g. "I have to go faster") or any kind of qualitative speed language	☐ Makes an incomplete quantitative statement about relative speeds of sliders (e.g., "you have to move twice as fast as I do")	☐ Uses the idea of steepness or slope to talk quantitatively about the relative speed of sliders, the slope of the line, or both (e.g., "this line is steeper than the last one, so you'll have to move twice as fast as last time," "this line has a slope of one, so we move at the same rate")	**Highest level checked** **Level 1: No boxes checked** Rating:______
Exploring mathematical relationships	☐ No explicit coordination of movement for making diagonal line (e.g., "you go to 7, I go to 3")	☐ Coordinates beginning of movement aloud (e.g., "3, 2, 1, go," "ready, set, go," "ready")	☐ Counts to coordinate movement of sliders	☐ Uses a more sophisticated coordination strategy, such as explicitly checking for intermediate spots on line (e.g., "we should both be on 5 now")	**Highest level checked** **Level 1: No boxes checked** Rating:______
Achieving mathematical goals	☐ Successfully completes challenge 1 with some accuracy (no diagonal lines)	☐ Successfully completes challenge 2 with some accuracy (diagonal lines with slope = 1)	☐ Successfully completes challenges 3 and/or 4 with some accuracy (diagonal lines with slope $\neq$ 1)	☐ Completes a planned free drawing (not random doodling) with some accuracy Number of level 2: ______ Number of level 3: ______ Number of level 4: ______	**Highest level checked** **Level 1: No boxes checked** Rating:______

Rating mathematical quantities for Drawing in Motion

- Level 1: No boxes checked.
- Level 2: Just ONE of the two boxes checked (either horizontal or vertical axis).
- Level 3: Both of the boxes checked (both horizontal and vertical axes).
- Level 4: Level 3 plus just ONE instance of describing direction or mathematical language. Describing direction and/or shape of line doesn't include directions to other visitors about how they should move (e.g., "go down to 4").
- Level 5: Level 3 plus MULTIPLE instances of describing direction or using mathematical language, using at least two different words.
- *Level 3 for mentioning one or more directional words but not verbalizing motion on both sliders.*

Drawing in Motion-specific coding instructions

- For achieving mathematical goals, "some accuracy" means that lines are close to pictures as intended. Horizontal and vertical lines go pretty much directly from one point to the next. Diagonal lines follow the general intent of the line slope. Using a horizontal and a vertical line to connect two points that are intended to form a diagonal line does not count as "some accuracy." Do-overs are fine. Being accurate on just SOME of the image is fine, as long as at least one diagonal line is drawn with some accuracy.
- For achieving mathematical goals, if a family skips all the challenges and just does free draw, the level is based on the difficulty of what they drew (i.e., if it had a diagonal line or not and whether diagonal lines had a slope of 1 or not). Drawing a curve automatically counts as "diagonal line with slope not equal to 1." If a family does two or more free draws, at least one of which would qualify as level 4, the group should be rated level 5.

Math Reasoning Behavior Checklist: *Designing for Speed* Coder initials:_______ Date:_______ Group #:_______

	Level 2	Level 3	Level 4	Level 5	Rating
Talking about mathematical quantities	☐ States order of wheels finishing but does not mention any quantities (e.g., "mine won")	☐ Mentions speed explicitly (e.g., "the yellow one is faster") ☐ Mentions time it took for wheels to finish explicitly, reading off the timer (even if read incorrectly) ☐ Mentions weight distribution on wheels			**(See below)** **Level 1: No boxes checked** Rating:______
Describing mathematical relationships	☐ Describes relationship in terms of color or some other non-mathematical description (e.g., "the yellow is fastest")	☐ Makes a statement about the relationship of weight position and speed on specific wheels but doesn't generalize to all wheels	☐ Makes a general statement about the relationship between the position of the weights and the speed of the wheel		**Highest level checked (max level 4)** **Level 1: No boxes checked** Rating:______
Exploring mathematical relationships	☐ Compares pairs of wheels and notes which one wins/goes faster but doesn't keep track	☐ Compares pairs of wheels, notes which one wins/goes faster, and keeps track across trials, either orally or written	☐ Notes how long a wheel took by reading timer and keeps track across trials, either orally or written (level 4 requires verbalizing or writing the timer readings)	☐ Describes both order of wheels in terms of speed/time and order of weight distribution across wheels	**Highest level checked** **Level 1: No boxes checked** Rating:______
Achieving mathematical goals	☐ Sets weights on adjustable wheel without justification or prediction statements	☐ Makes a prediction about how adjustable wheel will compare with another but doesn't note if it is accurate or not	☐ Sets weights, makes a prediction, and notes whether or not prediction was accurate		**Highest level checked (max level 4)** **Level 1: No boxes checked** Rating:______

Rating mathematical quantities for Designing for Speed

- Level 1: No boxes checked.
- Level 2: Level 2 box checked.
- Level 3: Mentions one quantity (one box checked).
- Level 4: Mentions two quantities (two boxes checked).
- Level 5: Mentions all three quantities (three boxes checked).
- *Level 2 not necessary for levels 3–5.*

Designing for Speed-specific coding instructions

- If both sides of the exhibit are being used, comparative statements may be made about the result of a single “race.” If only one side is being used, the comparison is based on the timer reading.
- For exploring mathematical relationships, “keeping track” of the comparison of speeds of the wheels must be explicit and can be either written or oral. If written, the time for at least two wheels—or the order of finishing of at least two wheels—must be noted in writing. If oral, the visitors must verbally and explicitly compare the time of at least two different wheels (e.g., “this one took 15 seconds and the last one took 13,” or “this one took 12 seconds, which is faster than the yellow”). If the facilitator is the one keeping track, this does not count.
- For exploring mathematical relationships, keeping track of the wheel times is counted towards family ratings if family members either write the numbers themselves or verbalize the times, which are then written by the educator. If the educator both says and writes the wheel times and the family does not refer to these times in any other way, this is not counted towards exploring mathematical relationships.
- For achieving mathematical goals, mark “N/A” if the interaction is not facilitated (i.e., greeting condition) and the challenge wheel not available.
- For achieving mathematical goals, visitor comments stated as questions are still be counted as predictions.