EMPower
extending mathematical power

Seeking Patterns, Building Rules

Algebraic Thinking

TEACHER BOOK

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EMPower Product Sampler Correlations for Algebra

EMPower Mathematics Topic Correlation Guide			
Seeking Patterns, Building Rules: Algebraic Thinking			
Book Description: Students use a variety of representational tools—diagrams, words, tables, graphs, and equations—to understand linear patterns and functions. They connect the rate of change with the slope of a line and compare linear with nonlinear relationships. They also gain facility with and comprehension of basic algebraic notation.			
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/ Topics Covered:**	**Pages: (Teacher Book)**
Opening the Unit	Seeking Patterns, Building Rules	• Personal **patterns described** and term 'pattern' explored for assessment purposes • **Algebra vocabulary** list initiated • **Prior algebra knowledge** assessed	pp. 1–16
Lesson 1:	Guess My Rule	• **Patterns/relationships between two variables** in a visual pattern • Expressing patterns in equation form	pp. 17–28
Lesson 2:	Banquet Tables	• Tracking table data • **Multiple representations of a situation** to predict outcomes	pp. 29–40
Lesson 3:	Body at Work–Tables and Rules	• Verbal and symbolic rule practice	pp. 41–54
Lesson 4:	Body at Work—Graphing the Information	• **Graph features** identified and compared • **Graphs generated** from tables and/or equations	pp. 55–64
Lesson 5:	Body at Work—Pushing It to the Max	• **Graph construction and connections** practiced • ***x-y* relationships** explored	pp. 65–76
Lesson 6:	Circle Patterns	• **Diameter and circumference relationship** explored • Rule and formula application	pp. 77–88
Midpoint Assessment	Using the Tools of Algebra	• **Production and interpretation of representations** assessed • **Symbolic notation** use assessed	pp. 89–94
Lesson 7:	What Is the Message?	• Translating equations • Equivalent expressions • **Coefficients** – meaning and representations	pp. 95–108

Lesson 8:	Job Offers	• Algebraic problem solving • Point of intersection • ***Y*-intercept**	pp. 109–122
Lesson 9:	Phone Plans	• Features of graphs • Matching representations • **Supporting decisions** with mathematical information	pp. 123–132
Lesson 10:	Signs of Change	• **Constant rate of change** identified and compared in representations	pp. 133–142
Lesson 11:	Rising Gas Prices	• **Linear and non-linear patterns**/rates of change compared	pp. 143–156
Lesson 12:	The Patio Project	• Algebraic knowledge applied	pp. 157–164
Closing the Unit	Putting It All Together	• Algebraic knowledge assessed	pp. 165–174

FACILITATING LESSON 9

Phone Plans

How do I know which plan is best for me?

Synopsis

Students piece together information about four long-distance phone plans and decide which plan best suits three customers with different needs. Previously, situations revealed patterns with a constant increase or decrease; here students also consider a situation where there is no change.

1. Pairs match ads, tables, graphs, and equations for four phone plans.
2. The class comes together to discuss graph and equation features that aided the matching process.
3. Pairs consider three consumer scenarios to decide which plan would be best and which would be worst for the customer, and justify their reasoning.
5. Everyone reflects on the representations, and the class discusses which tool they would use if faced with a similar situation.

Objectives

- Match graphs, tables, equations, and verbal rules by identifying the related features in each representation
- Connect the flatness of a horizontal line on a graph to a situation in which there is no change over time
- Use information from tables, graphs, rules, and equations to support consumer decisions

Materials/Prep

- Calculators
- Newsprint or transparencies
- Rulers
- Tape or glue

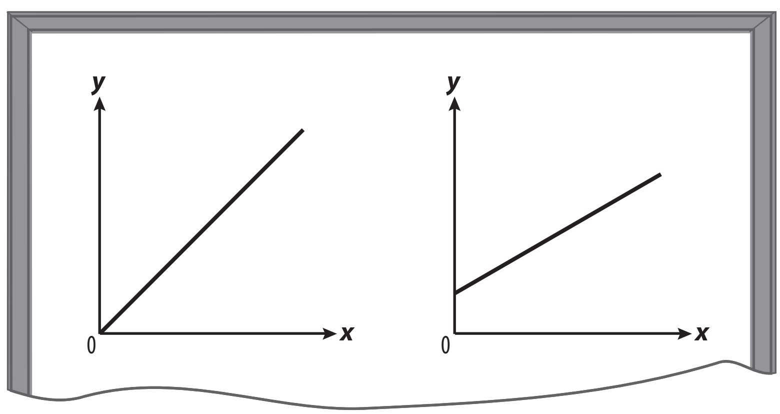
Make an enlarged version of each phone-plan graph (*Student Book*, pp. 114–15), on newsprint, or reproduce the graphs on transparencies.

Photocopy *Phone Plans*, *Blackline Master 6*, one for every pair of students. Cut the 12 pieces apart and put them in an envelope. (Students could do the cutting as well, but the scrambled pieces with unknown equations are more dramatic.)

Opening Discussion

(Optional: You might want to briefly review inequality symbols, which will be used in the phone-plan description. Write on the board: >, <, =. Ask volunteers to come to the board to read and write statements using one of the three symbols. Students share ways they remember the direction of the "is greater than" and "is less than" signs.)

Open by displaying a sketch of the salary graphs from *Lesson 8*.



Ask:

What on the graph told you about the starting bonus?

What on the graph told you who was getting a higher weekly rate of pay?

Say:

Today you will have four phone plans to compare in order to make some consumer decisions.

Activity 1: Phone Plans

Pair up students and distribute an envelope of cards (*Phone Plans*, *Blackline Master 6*) to each. Ask for a volunteer to read aloud the directions for *Phone Plans* (*Student Book*, p. 114). Allow everyone to work independently for 10 minutes, attaching the graphs to their matching representations. Observe which features draw students' attention and are used to help them connect representations.

Challenge those who finish quickly with an additional question:

Which monthly plan is the best for $50? Why?

Then ask students to pair up again to share their reasoning. Ask some pairs that come to agreement to write the equation, table, and ad for one plan on the enlarged versions of the graphs that you prepared earlier.

Draw the class together to address each plan. Pose questions that illuminate the graph features:

What tells you these go together?

Where do you see the table data in the graph?

Where do you see the equation in the graph—the coefficient, or the constant number added?

Invite students to the board to demonstrate how they made the connections. In particular ask:

Phone Plan A

Why does this line start here? (Point to the origin.)

Phone Plan B

Why does this line start here? (Point to (0, 5).)

Which graph is steeper—Plan A's or Plan B's? What does that tell you?

Phone Plan C

How is this graph different from those for Plans A and B?

Why is part of the graph flat? (Encourage references to the tables and equations to support statements made.)

What is happening at this point? (Point to (1,000, 40).)

What would the ad say if the graph looked like this? (Draw a flat line: $y = 40$.)

Phone Plan D

Compare the graphs of Plan D and Plan C.

Clear up any confusion about the equations by asking questions such as

Why do you subtract 1,000 from *M* in the equation for Plan C and subtract 250 from *M* in the equation for Plan D?

Ask for some examples to emphasize the connection between the situation described verbally in the ad and symbolically in the equation. Then ask about the rate of change:

Which plan shows the fastest rate of increase in cost for any period of time? What tells you that information?

Expect to hear that some see the rate of increase in the graph and others in the equation for Plan C.

Heads Up!

The conversation in this first activity can get very involved. Move on to the next activity. In the summary, continue discussing connections between representations.

Activity 2: It Would Depend on the Person

Turn to *It Would Depend on the Person* (*Student Book*, p. 116).

This activity uses the phone-plan representations to support a consumer decision. Students will need all four re-pieced ads to come to a good recommendation. Three kinds of customers are portrayed.

Suggest students count off: 1, 2, 3, 1, 2, 3 … to determine the number of the problem they will work on. Allow them to work independently for about 15 minutes.

When most students have completed the problem, ask all those with the same numbers to form groups to share their answers and the ways they made their decisions. Allow time for each group to come to a common agreement for a phone plan for its customer. Then ask a spokesperson from each group to make a persuasive recommendation to their customer as to the *best* and the *worst* plan for that person. Every statement should be justified using at least one type of representation.

If the following have not been answered by the group's spokesperson, ask:

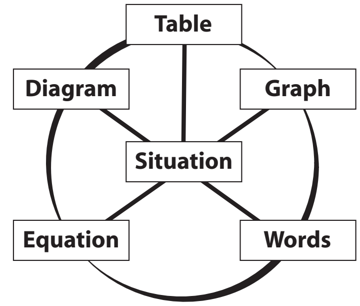
Which representation(s) did you use to help you decide?

What information in that representation swayed your decision?

How is your decision supported by another representation?

Assign Problem 4 to groups who finish early or for homework.

Summary Discussion



Discuss the different representations everyone used to help make decisions and what information was gleaned from each representation. Ask:

Which type of representation would you turn to first if you had to make this kind of decision again? Why?

Encourage specificity in responses, using today's work for points of reference. Refer students to their *Reflections* section (*Student Book*, p. 162).

Practice

I Am Changing the Rules!, p. 117
Gives students an opportunity to adjust one of the plans and its representations accordingly.

Symbol Sense Practice

Greater Than, Less Than, p. 119
Asks students to express conditions in mathematical notation using inequality symbols.

Solving Two-Step Equations, p. 120
Suggests some ways to solve equations in the form $ax \pm b = c$, using a simple strategy.

Extension

Looking at Four Graphs, p. 122
Asks for justification based on intersections of the four phone-plan graphs.

Test Practice

Test Practice, p. 123

Looking Closely

Observe whether students are able to

Match graphs, tables, equations, and verbal rules by identifying the related features in each representation

If students gravitate toward certain numbers and make decisions based on them—such as the number 1,000, which surfaces in the equation, the table, the graph, and the ad for Plan C—ask where the number occurs in other representations. What explains the flatness of this line? What tells you the graph starts on the y-axis?

Look for informal connections that arise, such as

- When Out numbers increase in a table as the In numbers increase, the line goes up to the right.
- When Out numbers stay the same in a table as the In number increases, the line is flat.
- The number multiplying x in an equation is the difference between the y numbers in the table.
- When the graph does not start at the origin, a number is added to another in the equation.
- If one line is steeper than another, the cost is increasing at a faster rate in the first line.

Connect the flatness of a horizontal line on a graph to a situation in which there is no change over time

Label a few points on the graph, and connect these to numbers in the tables and the ads, if students do not understand what the flatness indicates.

Ask students to think of other situations where there is no change over time (flat rates). For instance, if a heart-monitor graph shows a **flat line**, what does that mean?

Use information from tables, graphs, rules, and equations to support consumer decisions

Ask students what each person would be looking for in a phone plan. Students should be able to explain their decisions by referencing information presented in the various formats.

WHAT TO LOOK FOR IN *LESSON 9*	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
Representations Graphs • Explains reasons for shape of each graph				
Connections • Matches representations with confidence • Flexibly moves among representations, connecting key features • Explains how table information shows up on each graph • Explains how features of the equations connect to the graph and the table				
Problem Solving • Uses information in the representations to solve problems and support conclusions				

Rationale

This lesson focuses attention on the different representations of a consumer scenario, and the capacity of algebra to simplify complicated situations. Learning to use the tools of algebra efficiently helps students evaluate situations that involve more than one set of conditions.

Math Background

In this lesson, the slant of a constant function, which appears in a graph as a flat line, is considered (a constant function with zero slope). In more formal algebra, the disappearance of x in such situations is explained by the fact that $y = mx + b$, and $m = 0$. Here, however, students focus on the constancy of the output, no matter the value of the input.

Notation is introduced for a situation where the rule changes at a certain point. [When $M \leq 1{,}000$, $C = 40$. When $M > 1{,}000$, $C = 40 + 0.35\ (M - 1{,}000)$].

Context

Phone plans are evolving, and many people have to make decisions about cell-phone and long-distance plans. The pricing structure for cell-phone plans used in this lesson was common in 2004. You might want to consider other pricing scenarios that have evolved since.

Facilitation

Making the Lesson Easier

Start with matching the graphs and tables first. Then match the ads with the equations.

Making the Lesson Harder

Change aspects of the graph, and ask people to predict how the changes might affect the equation and vice versa.

Gather some advertisements, such as grocery ads from three stores, and ask students to develop representations for one product.

Activity 2: It Would Depend on the Person motivated students to bring their everyday decision-making skills to bear when choosing the phone plan best suited to each person's particular phone habits.

For the final question on their own best phone plan, some students were sure of their monthly usage—400 minutes, for example—and compared plans accordingly:

A = $ 28 per 400
B = $ 30 per 400
C = $ 30 per 400
D = $ 40 per 400

The flat rate of $40.00 for 1,000 minutes appealed to many:

Plan C will be better for me, I make call anytime and 1000 Minutes is a lot of Minute at the rate of $40- per month.

Barbara Tyndall used a two-part rubric ("What I notice" and "My interpretation and next steps" to help her students examine their work. Below are examples of two students' written work from Practice: I Am Changing the Rules!, along with Barbara's written comments.

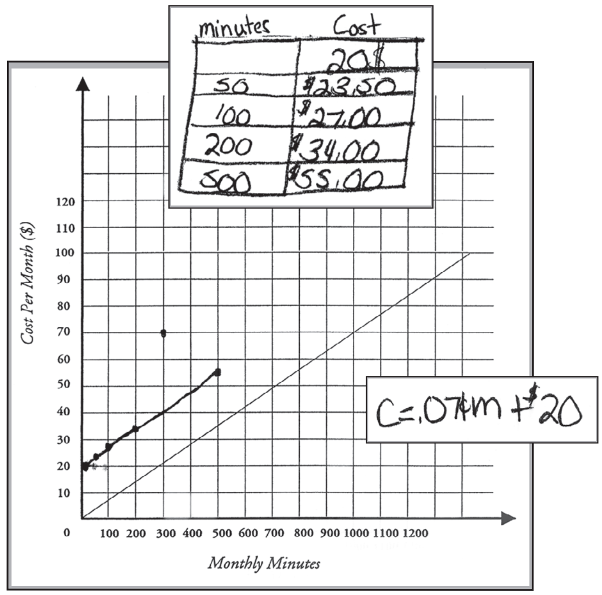
What I notice:

- Can write an equation to go with the verbal description.
- Creates a table that is accurate and reflects the initial cost.
- Correctly graphs the equation for the new rule, new equation, new table!

My interpretation and next steps:

- Can be alerted to correct notation ($0.07 or 7¢, not $0.07¢).
- Could put "0" in minutes column for first entry (student had erased the zero).
- In response to my asking what the student notices about the two line graphs, he or she mentions "constant rate of change" and "slope."
- Student is ready to go to next lesson!

Student 1 Response to I Am Changing the Rules!



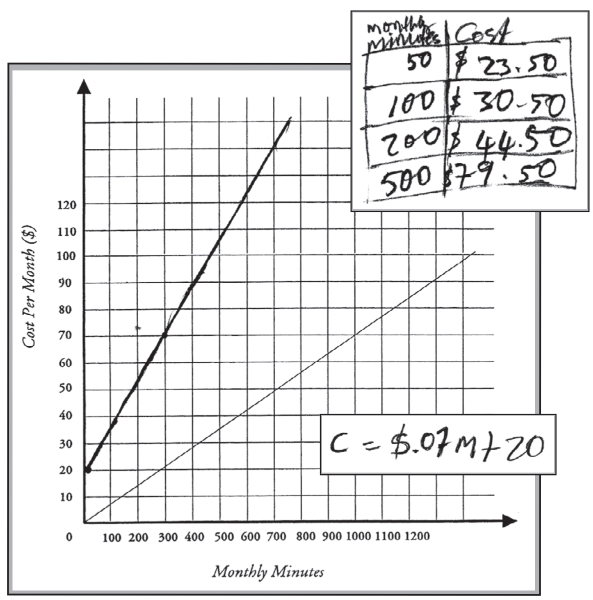
What I notice:

- Writes a new equation that shows the new rule.
- Correctly calculates the cost for 50 minutes, but subsequent costs are incorrect in the new table.
- Graph starts out at the correct point (0, 20), but does not accurately reflect the data in the table.

My interpretation and next steps:

- Understands the connection between the verbal description and the equation.
- Can use the equation to calculate the cost between the first value (of 50 min.) for time, but seems to use a different pattern to complete the table.
- May have difficulty estimating the values on the scale.
- Needs more practice before going to the next lesson.
- Should be urged to check that the values in the graph all follow the rule described.

Student 2 Response to I Am Changing the Rules!



Lancaster-Lebanon Intermediate Unit/13 Career Link, Lancaster, PA

Blackline Master 6

Phone Plans

Cellular wireless

7 cents per minute

No Monthly Charge!

Z-phone

250 Anytime Minutes

$15 per month

(30 cents each additional minute)

$C = \$5.00 + \$0.05M$

$C = \$0.07M$

Tel-Mobile

$5.00 Monthly Fee

5 cents/minute

NuWave Wireless

1,000 Anytime Minutes for $40.00/Month

$0.35 each additional minute

Monthly Minutes	Cost
50	$3.50
100	$7.00
200	$14.00
500	$35.00

Monthly Minutes	Cost
50	$7.50
150	$12.50
250	$17.50
350	$22.50

Monthly Minutes	Cost
100	$15.00
150	$15.00
250	$15.00
300	$30.00

Monthly Minutes	Cost
500	$40.00
1,000	$40.00
1,100	$75.00
1,200	$110.00

for $M \leq 250$, $C = \$15$

for $M > 250$, $C = \$15 + \$0.30\,(M - 250)$

for $M \leq 1{,}000$, $C = \$40$

for $M > 1{,}000$, $C = \$40 + \$0.35\,(M - 1{,}000)$

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LESSON 9

Phone Plans

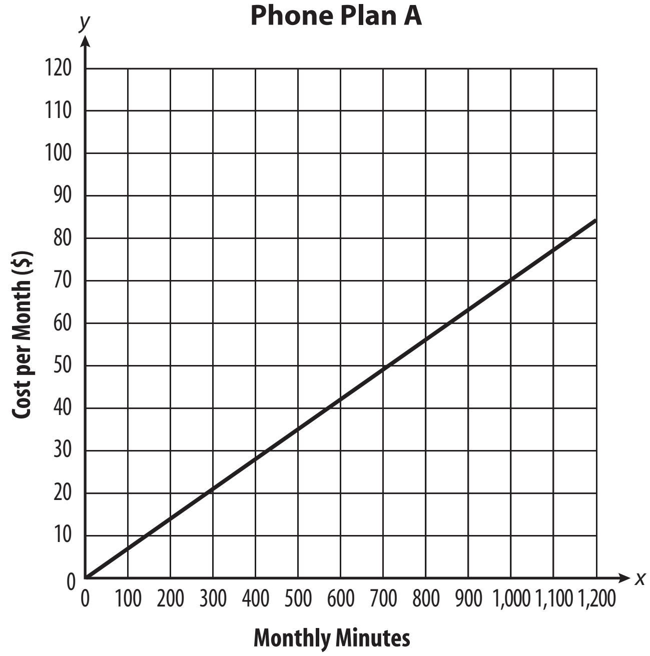
How do I know which plan is best for me?

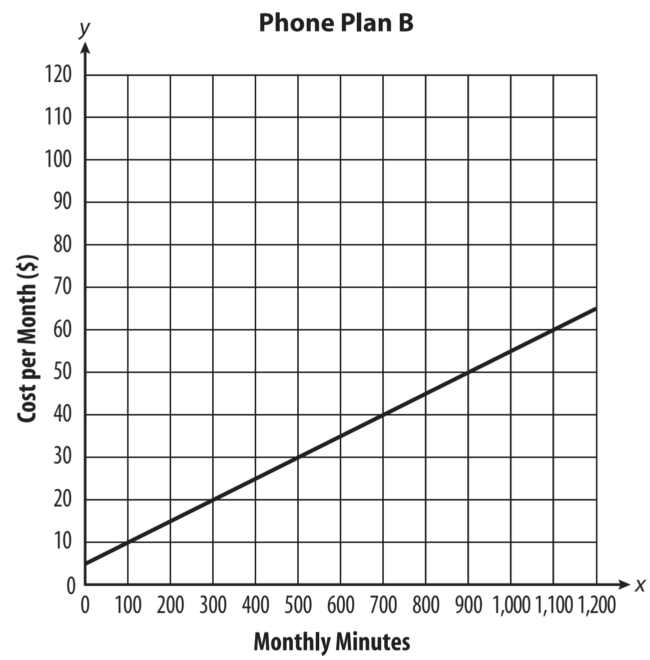
Ads are everywhere—luring us to choose a long-distance phone plan or to take a loan on credit, for example. It is not always easy to figure out which company offers the best deal. Your algebra tools will help you see more clearly which deal is best.

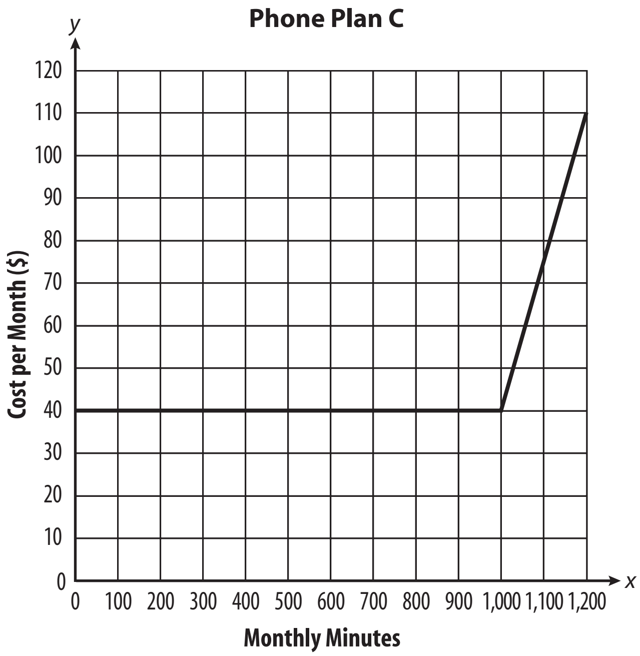
You will piece together information about four phone plans. To find the best deal, you will have to make sense of tables, graphs, words, and equations. Then you will think about the advantages and disadvantages of the plans for particular customers.

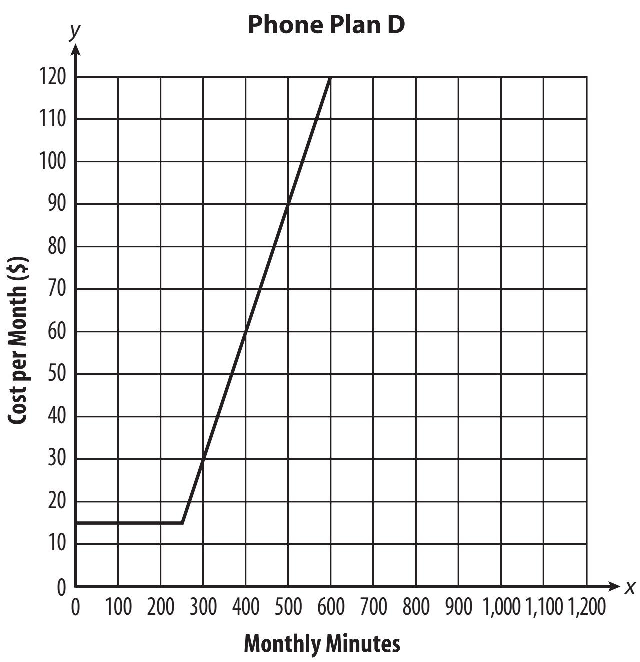
Activity 1: Phone Plans

Four ads for cellular and long-distance phone plans came in the mail today, but the ads got ripped up. Can you put the pieces back together? Your teacher will give you all of the words, tables, and equations from the ads. Reattach them to their matching graphs.

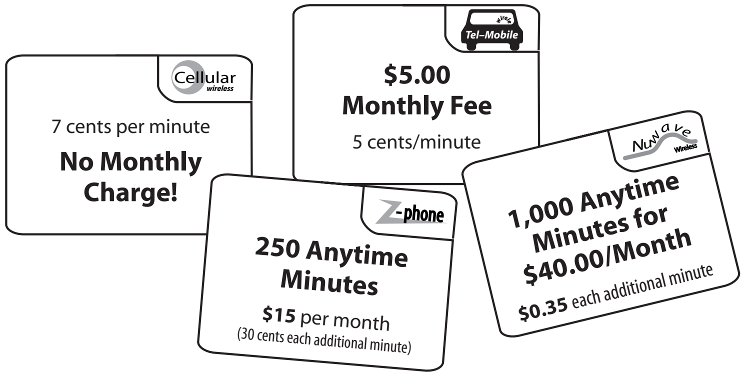





Phone Plan C
y
120
110
100
90
80
70
60
50
40
30
20
10
0
Cost per Month ($)
x
0 100 200 300 400 500 600 700 800 900 1,000 1,100 1,200
Monthly Minutes


Phone Plan D
y
120
110
100
90
80
70
60
50
40
30
20
10
0
Cost per Month ($)
x
0 100 200 300 400 500 600 700 800 900 1,000 1,100 1,200
Monthly Minutes

Activity 2: It Would Depend on the Person



Use the pieced-together advertisements to answer the following questions about which plans are best and worst for specific customers:

1. Mary Jane loves to talk to her family, many of whom live out of state. She wants as many minutes as she can get, but definitely does not want to pay more than $50.00. Which plan should she choose? Why? Which plan would be the worst for her? Why?

2. Jenny, who lives in New York City, talks to her best friend in Miami every night for about a half-hour. Other than that, she makes very few long distance calls. Which plan is best for her and which is the worst? Why?

3. Tricia wants to get a phone she will only use in emergencies, for instance, if her car breaks down. Which plan is best for her and which is the worst? Why?

4. Which phone plan would be the best for you? Which would be the worst for you? Why?

Practice: I Am Changing the Rules!

You are the new Chief Executive Officer (CEO) of Cellular Wireless that has advertised Plan A, and you think it is time to change the rules.

The Old Rule

Cellular wireless

7 cents per minute

No Monthly Charge!

The New Rule

Cellular wireless

Only 7 cents per minute

Unlimited Minutes!!! *

(*$20.00 one-time joining fee)

Change the old equation, table, and graph to go along with your new rule for the first month.

The Old Equation

C = $0.07M

The New Equation

____

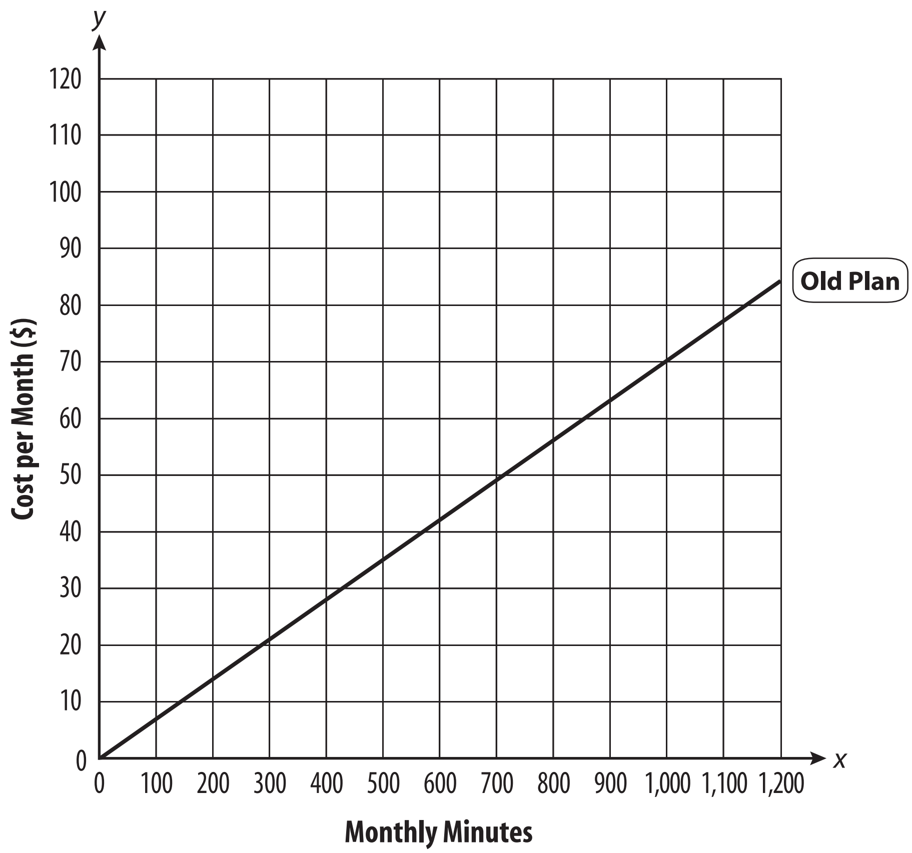
The Old Table

Monthly Minutes	Cost
50	$ 3.50
100	$ 7.00
200	$14.00
500	$35.00

The New Table

This is a graph of the old plan.

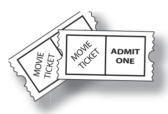
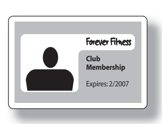
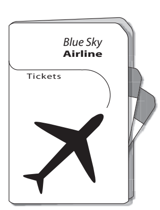
Graph the new plan here in another color.



Symbol Sense Practice: Greater Than, Less Than

Math Symbol	In Words
=	equals
>	is greater than
<	is less than
≥	is greater than or equal to
≤	is less than or equal to

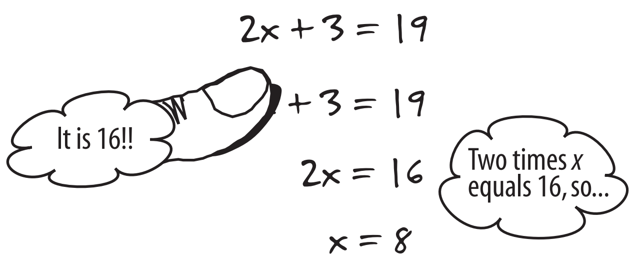
Use the above math symbols to rewrite the advertisements below where cost depends upon age. Let C stand for cost, and let A stand for age.

Advertisements		Math Symbols
1. Movie tickets		
	Under 12 years: $5.00	When A < 12, C = $5.00
	12 and over: $9.00	When A ≥ 12, C = $9.00
2. Coffee at a diner		
	Cup of coffee: $1.25	
	Senior citizens (65 and over): $0.05	
3. Club membership		
	Under 21: free	
	21 and over: $10.00	
4. Airline tickets		
	Five and over: $250	
	Under five years: half price	

Symbol Sense Practice: Solving Two-Step Equations

One easy way to solve two-step equations is to cover part of the equation using the "finger cover-up" method.

Faced with an equation such as $2x + 3 = 19$, put your finger over the "$2x$" part of the equation like this:



Then ask yourself: What number would I add to 3 to get 19? You would add 16. That means that 2 times the missing number (x) must equal 16. The missing number must be 8!

Does it work? Replace the "x" with an 8 to see whether it makes sense:

$2(8) + 3 = 19$

It works! Try this method with the problems below. Remember to rethink your questions when you are subtracting, not adding, a number.

Balance the amounts. For Questions 1–10, find the missing number using the cover-up method.

1. $2x + 10 = 20$ $x =$ _______

2. $5x + 9 = 49$ $x =$ _______

3. $2x - 50 = 150$ $x =$ _______

4. $4x - 5 = 15$ $x =$ _______

5. $4x + 20 = 100$ $x =$ _______

6. $10x + 900 = 1,000$ $x =$ _______

7. $100x - 50 = 150$ $x =$ _______

8. $8x + \frac{1}{2} = 56.5$ $x =$ _______

9. $\frac{x}{2} + 7 = 27$ $x =$ _______

10. $\frac{x}{10} + 8 = 88$ $x =$ _______

For Questions 11–15, write the equation first, and then solve it using the finger cover-up method.

11. If you double a certain number and add 3, you will get 7.

The equation is ________________________. $x =$ _______

12. Multiply a certain number by 5 and add 3 to get 28.

The equation is ________________________. $x =$ _______

13. If you multiply a certain number by 1,000 and add 250, you will get 6,250.

The equation is ________________________. $x =$ _______

14. Multiply a certain number by 7 and subtract 4 to get 59.

The equation is ________________________. $x =$ _______

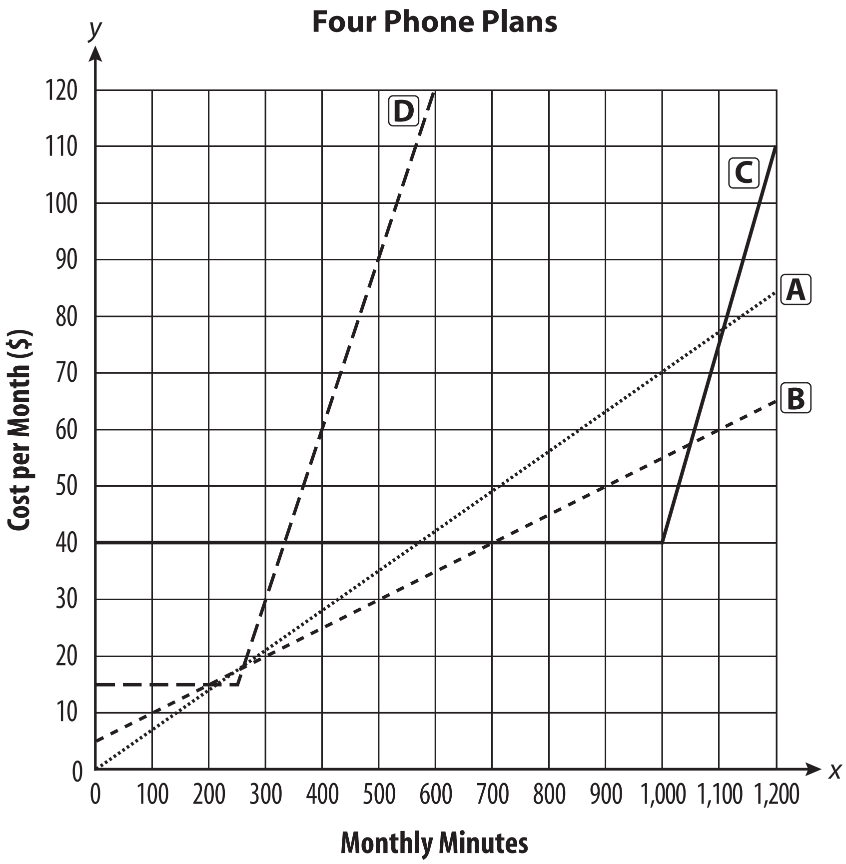
15. Half a certain number minus 10 is 41. Find the number.

The equation is ________________________. $x =$ _______

Extension: Looking at Four Graphs

Juania says: "I really do not think it makes much difference for me. On Plans A, B, and D, I will pay about the same … But Plan C is definitely out."

What do you think her calling pattern is? Use the graph to support your conclusion.



Test Practice

1. Which is a true statement?

   (1) $15 + 10 < 20$

   (2) $20 > 5 + 10$

   (3) $4(9) > 9(4)$

   (4) $4 + 9 > 9 + 4$

   (5) $20 + 5 < 15$

2. Which equation below would have the steepest graph?

   (1) $y = 2x + 20$

   (2) $y = 2x + 10$

   (3) $y = x + 1{,}000$

   (4) $y = 3x + 5$

   (5) $y = x - 1{,}000$

3. In which equation is 40 the value of x?

   (1) $x + 5 = 35$

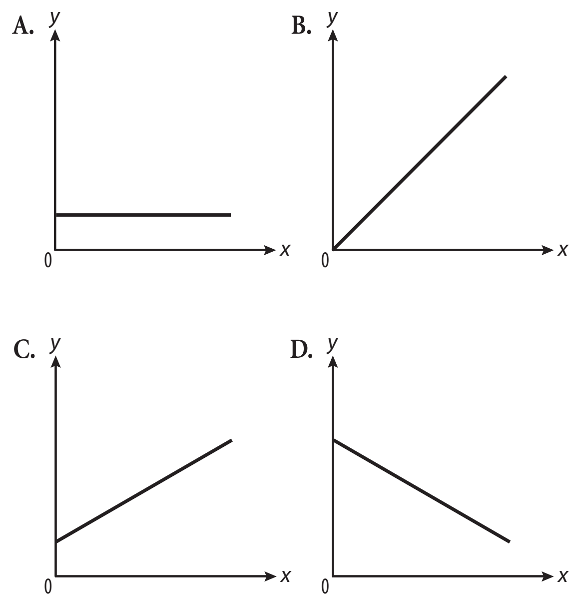
   (2) $x - 5 = 45$

   (3) $45 = 2x - 5$

   (4) $x = 45 - 5$

   (5) $\frac{x}{2} = 90$

4. Which graph might represent the equation $y = 10$?



   (1) A

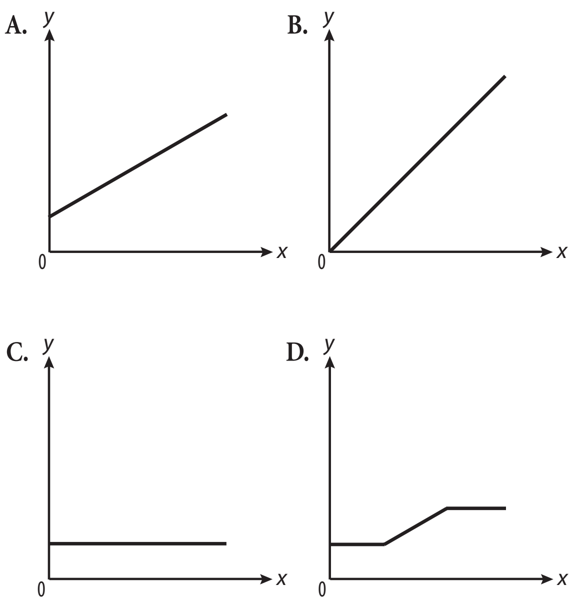
   (2) B

   (3) C

   (4) D

   (5) None of the above

5. Pedro likes to talk every day as much as he can on the phone. Which graph below represents a phone plan he would want? Cost of service is listed on the *y*-axis and minutes used on the phone is listed on the *x*-axis. The scales and intervals are the same on all graphs.



(**1**) A

(**2**) B

(**3**) C

(**4**) D

(**5**) None of the above

6. If $x = 5$, what is the value of $3x - \frac{1}{2}$?