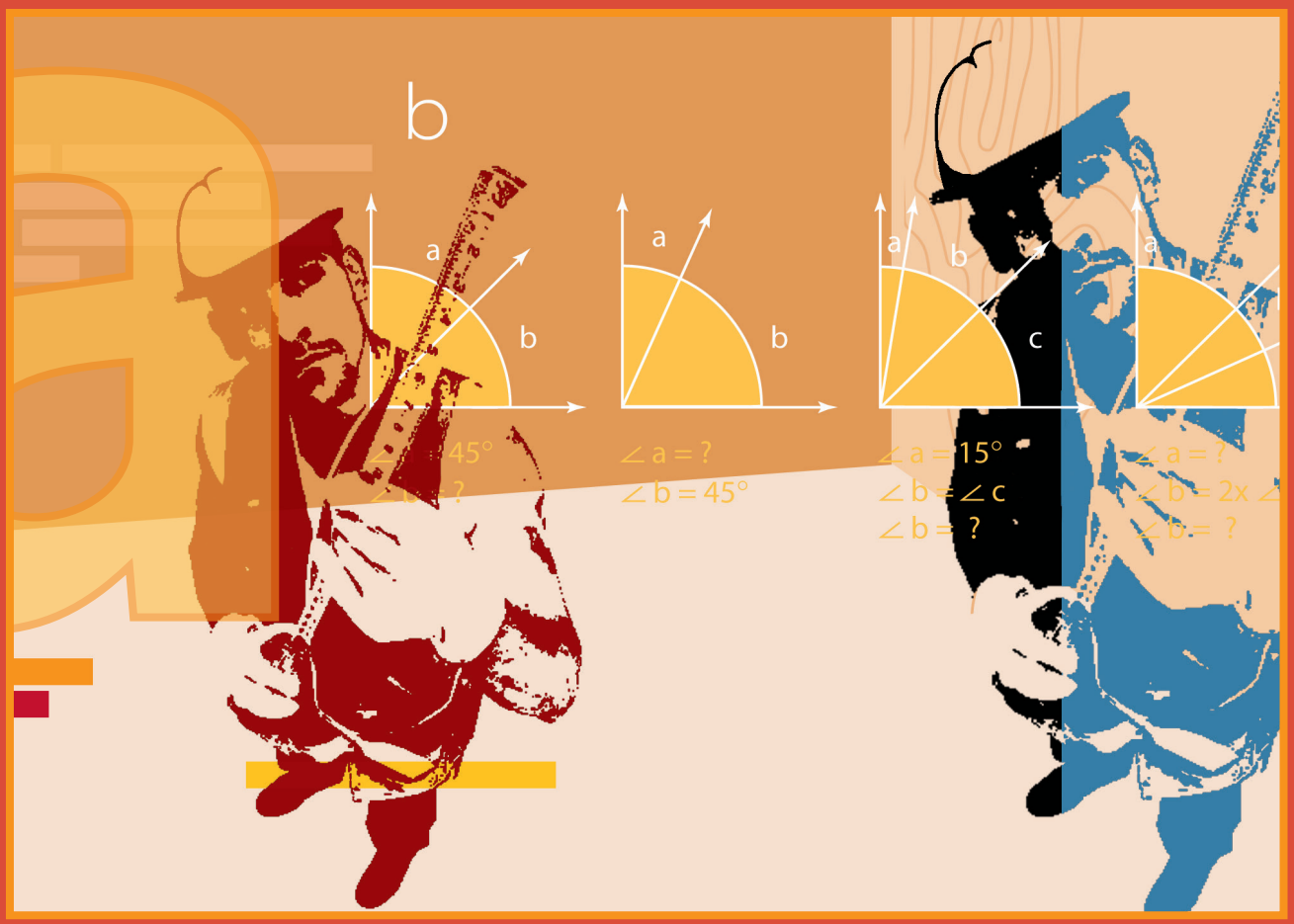
EMPower
extending mathematical power

Over, Around, and Within

Geometry and Measurement



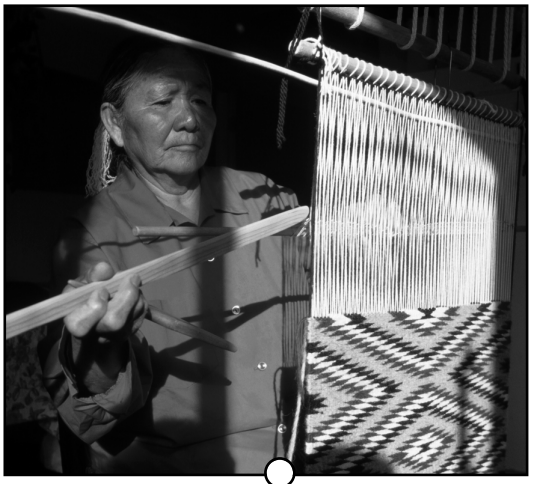
TEACHER BOOK

EMPower Product Sampler Correlations for Geometry

EMPower Mathematics Topic Correlation Guide			
Over, Around, and Within: Geometry and Measurement			
Book Description: Students explore the features and measures of basic shapes. Perimeter and area of two-dimensional shapes and volume of rectangular solids provide the focus.			
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/ Topics Covered:**	
Opening the Unit	Geometry Groundwork	• **Shapes** identified and sketched • **Angles** introduced • **Geometry vocabulary** list started • **Prior Geometry knowledge** assessed	
Lesson 1:	Sharing Secret Designs	• **Two-dimensional shape characteristics** identified • **12 Basic Geometric Shapes** identified and described	
Lesson 2:	Get It Right	• **Angles identified and described** with conventional notation • **Right angles** introduced • **Angle measurements** estimated with 90° benchmark and determined precisely with protractors	
Lesson 3:	Get It Straight	• **Straight (180°) angles** explored • **Sums of angles** in triangles and rectangles established	
Lesson 4:	Giant-Size	• **Similar shapes** identified and described • **Length and width dimensions** introduced and measured • **Perimeters** determined by adding	
Interim Assessment 1	Shapes and Angles	• **Attributes of shapes' and angle measurements'** knowledge assessed	
Lesson 5:	Line Up by Size	• **Area and perimeter** distinguished	
Lesson 6:	Combining Rectangles	• **Rectangle area calculated** in square centimeters • **Composite shapes'** areas and perimeters compared	

Lesson 7:	Disappearing Gridlines	• **Formulas for area and perimeter** derived • **Missing dimension values** determined • **Area of a right triangle** calculated	
Lesson 8:	Conversion Experience	• **Standard English Units** introduced • **Linear unit conversions** established	
Lesson 9:	Squarely in English	• **Square units** – square inches, feet, and yards constructed and connected with area measure • **Square unit conversions** established	
Lesson 10:	Scale Down	• **Scale drawings** made and steps for scaling analyzed	
Interim Assessment 2	A Fresh Look	• **Area, perimeter, measurement, and scale** knowledge applied and assessed	
Lesson 11:	Filling the Room	• **Volume** explored as capacity • **Third dimension – height** becomes apparent	
Lesson 12:	Cheese Cubes, Anyone?	• **Cubic inch** introduced then used to measure volume • **Volume formula** derived	
Lesson 13:	On the Surface	• **Surface area and volume** compared • **Surface area and shape relationship** generalized	
Closing the Unit	Design a Box	• **Geometry and measurement knowledge** applied and assessed	

Facilitating Opening the Unit: Geometry Groundwork



Is there geometry in your life?

Synopsis

This session assesses students' familiarity with basic concepts and vocabulary associated with geometry and measurement. The activities establish a baseline by extending a variety of opportunities for students to interact with and comment on aspects of shapes, including dimensions, angle size, perimeter, area, and volume.

1. Students complete a Mind Map, recording their ideas and associations with geometry and measurement.
2. Students complete a self-assessment based on several problems.
3. Students discuss the geometrical shapes in homemade objects.

Objectives

- Demonstrate prior knowledge of geometry
- Construct an angle demonstrator and record an angle
- Keep a record of vocabulary useful to the study of geometry
- Identify and sketch shapes

Materials/Prep

- Brad fasteners
- Homemade objects and other objects with interesting shapes, such as picture frames, hand-sewn clothes, baked goods, and musical instruments
- Newsprint
- Paper
- Paper-clip boxes
- Rulers marked with centimeters and inches
- Scissors
- Shoe boxes

Ask students to bring homemade objects of their own to class.

Copy angle demonstrators (*Blackline Master 1*) on heavy paper, and cut out two strips per student. This adjustable tool allows students to replicate angles of different sizes.

Make copies of the *Initial Assessment*, p. 161, one for each student.

Opening Discussion

Begin by asking everyone to estimate sizes with their hands:

How big is a foot? A centimeter?

Look around; do we agree?

Provide an overview of the unit, and set the tone for the class. Tell students:

Measurement is key to exploring ideas of geometry, so we will start there. In this class, frequently there will be more than one right answer and many ways to find the answer, yet there also will be times when only one answer will be right.

A centimeter is an example of a standard metric measurement agreed upon by people around the world.

Give some examples of the advantages of standard units of measure:

Every city has a Department of Weights and Measures to monitor and enforce standards. For example, when you buy a gallon of gas, you are assured that every pump at every gasoline station will measure an exact gallon.

Nearly everything constructed by human beings began as a drawing. In this unit, you will learn the geometry useful for planning and designing. You will measure, draw, and plan on paper (in two dimensions) and build, estimate, and measure solids (in three dimensions).

Activity 1: Making a Mind Map

The Mind Map is the first artifact detailing students' prior knowledge of geometry and measurement. By the end of *Opening the Unit*, you should have several artifacts:

- Mind Map (*Activity 1*)
- Students' *Initial Assessment* (*Activity 3*)
- Labeled sketches from *Activity 4: Homemade Objects*
- Writing samples from *Practice: Using Geometry*

Start the Mind Map by writing "Geometry" and "Measurement" on the board. Explain the Mind Map concept:

Mind Maps are an effective way to generate information on a topic when you want to get ideas down quickly. As you brainstorm, you begin to organize ideas by making connections. Remember: When you are brainstorming, there are no wrong answers.

Ask:

Where do you see measurements in your everyday life? What do you know about them?

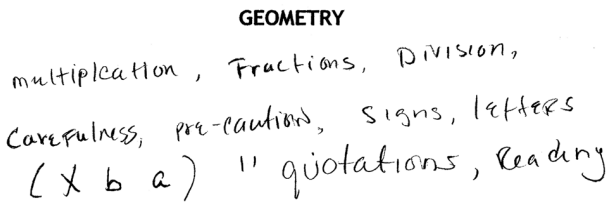
List some types of measurement units.

What do you know about geometry?

Write down a few ideas. Model clustering them, linking related ideas with lines, and then ask students to record their own ideas on p. 152 of the *Student Book*. After a few minutes, ask students for examples from their work.

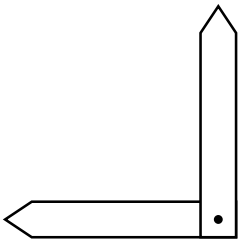
As terms related to geometry and measurement arise, start a class vocabulary list. Do not worry about formal definitions; use what students themselves say. You will want to add and refer to this word list, so write it on newsprint. Students can take notes in the *Vocabulary* section of their books, starting on p. 149.

This is an example of one student's mind map:



Activity 2: Making an Angle Demonstrator

Give every student two strips of paper (from *Blackline Master 1*) and a brad fastener. Model making an angle demonstrator. Place one strip on top of the other so the two dots are aligned. Put a fastener through the two dots so the parts can open and close like a mouth. (Directions are also in the *Student Book*, p. 3.) Show students how to use the angle demonstrator by holding it against an angle (for example, the corner of the board); trace the inside lines of the angle demonstrator onto a piece of paper. Tell students they will use their angle demonstrators in the *Initial Assessment* and again in a later lesson.



Activity 3: Initial Assessment

Because the *Initial Assessment* can look intimidating, it is recommended that you show the tasks on an overhead projector or otherwise display them for the whole class to see. In addition, distribute a copy of the *Initial Assessment* to every student.

Then explain that this is a test where students do *not* solve the problems. Instead they are to check off "Can do," "Don't know how," or "Not sure" in response to whether they feel they can solve the problem. Tell your students that the results will help you estimate the amount of time they may need on different topics covered in the unit, such as measurement, area, perimeter, and volume.

Show the tasks on the overhead projector. Remind students not to solve the problems, but to assess their confidence in their ability to solve them. Students can read silently or you can read to them. Keep the momentum going, reading the problem and then quickly moving on to the next one.

Collect the assessments and analyze them later.

Heads Up!

If most of your class indicate they can do all of the tasks, and their written work on *Using Geometry* is of high quality, photocopy the *Initial Assessment,* and at the next class meeting ask students to complete the tasks. See *Checklist* (p. 7) for information on instructional decisions.

Activity 4: Homemade Objects

Every student needs an object to complete this activity. There are two options:

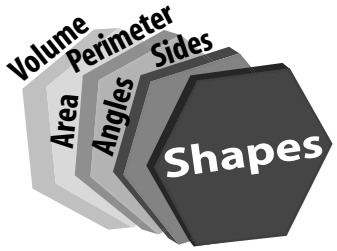
- Students can bring homemade objects to class.
- Students can choose an item from your collection.

Refer students to *Homemade Objects*, *Student Book*, p. 3. Give them time to sketch, identify, and label shapes *within* the object. Some objects will lend themselves better to this activity than others. Convey to students that you do not expect their representations to be perfect.

Model describing how you made one of your own objects. Include geometry terms and the vocabulary of measurement in your description. Then ask students to describe the shapes within their objects.

Summary Discussion

List terms students would like to remember or know more about based on discussions of objects they made or the *Initial Assessment*. If there is time, orient students to their books. They should answer the *Reflection* questions (p. 152). Also have them read through *Unit Goals* (p. 8).



Practice

Using Geometry, p. 5
This is the initial written assessment for the unit. It will be useful to have a sample of each student's work, explaining how he or she sees and uses geometry and measurement.

Seeing Geometry, p. 6
Students choose another object, sketch it, and describe its geometric properties and measurements.

Extension

Name the Shapes, p. 7

Looking Closely

Observe whether students are able to

Demonstrate prior knowledge of geometry

Do students have experience with this area of mathematics? Do they seem curious?

Construct an angle demonstrator and record an angle

Constructing a measurement tool will help students navigate through the activities of this unit. In addition, it will give you insight into their facility with constructing, aligning, and tracing—skills they will call on as they find area, perimeter, and volume, and record angles. Take note of students who seem adept at this activity; later you can assign students to work in pairs in ways that nurture emerging learners and challenge more advanced students. If some students have difficulty constructing the tools, pair these students with those who have succeeded and can offer help.

Keep a record of vocabulary useful to the study of geometry

Rather than consulting a dictionary, try to piece together definitions based on students' prior knowledge. These definitions can be refined over time.

Do English language learners know geometric terms in their own languages? Ask them to record these as well.

Identify and sketch shapes

Can students identify and name the overall shape of objects? Can they identify and name shapes within objects? Direct students to compare the attributes of shapes, such as length and position of sides and the number and shape of angles (e.g., wide-open or pointy).

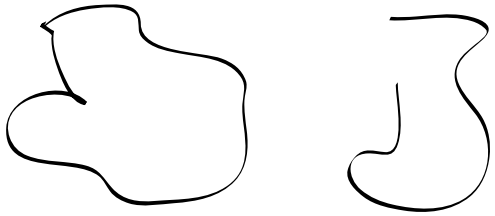
WHAT TO LOOK FOR IN *OPENING THE UNIT*	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
Concept Development • Identifies and names overall shapes of objects • Identifies and names shapes within objects • Compares shapes with other shapes • Records angles using angle demonstrator				
Expressive Capacity • Uses appropriate vocabulary when describing and comparing shapes • Describes shapes within objects with precision				
Use of Tools • Constructs angle demonstrator • Uses angle demonstrator				
Background Knowledge • Is familiar with geometric shapes • Has vocabulary for different shapes, angles, and geometric properties, e.g., area, perimeter, and volume				

Rationale

This lesson is an invitation for students at all levels to discuss shapes, measurements, and the ways in which geometry and measurement contribute to our ability to construct and describe objects. Constructing a measurement tool helps students internalize a sense of angle size. EMPower builds on students' prior knowledge; therefore, objects of students' own making are an ideal starting point.

Math Background

Recognizing the basic shapes—rectangles, squares, circles, and triangles—is critical to the study of geometry and measurement. Observing the characteristics of a shape, such as the number of angles or sides, their relative sizes, and the sides' relative positions, allows us to tap into the established knowledge base about the shape, for example, how to find the measurement for a missing side or angle, or the formula for area.



The world is full of squiggly lines and amorphous, incomplete, or open shapes. Open shapes are problematic when determining area. With no clear boundaries, open shapes lead to questions about infinity. Amorphous shapes present a challenge of a different kind. Finding the area of a shape with curves demands making approximations using rectangles, some of which are tiny, to fill in the shape. In calculus, this is done by finding the area under curve through integrals. However, knowing how to find the area of rectangular, triangular, or hexagonal shapes is the first step toward arriving at an estimate of the area or volume of an amorphous shape.

Facilitation

To promote the communication central to mathematics, ask students to interview each other about their homemade objects. Post interview questions such as

- **What steps did you take to make the object?**
- **What geometric shapes are part of your object?**
- **What did you measure?**
- **What were the most difficult and enjoyable parts of making the object?**

In some classes, interviewing can be difficult, given the English-proficiency level of the students. Further simplify the task by giving sentence starters, e.g., "I have a _____; I made it by _____."

OPENING THE UNIT IN ACTION

Teachers and students call on their powers of observation.

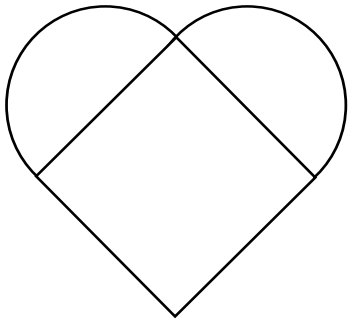
Two of my students seemed to wish they had stayed home today, but as they started explaining how they made their objects and measured them, they became engaged in the activity. I noticed one student did not know how to read a ruler. "Rectangle" was not in his vocabulary; he called the cereal box a square and measured the sides to see whether they were the same. The other reluctant student finished drawing his shapes quickly, but the longer he had to wait for his partner to finish drawing, the more he looked at his object, and the more detail he added to his drawing. By the time these two students switched objects to see what the other had missed, the student who was initially resistant was noticing all sorts of things about his partner's object. He tapped into great powers of observation, and then he suddenly looked at the ruler he could not read and started asking questions. He actually figured out ¼ , ½ , and ¾ inches.

Phyllis Flanagan
Rock Valley College Adult Education Center, Rockford, IL

One teacher's homemade item sparks discussion that calls on shape names.

The night before I taught this lesson, I decided to make a heart-shaped cake since it was close to Valentine's Day. Before I went into class, I was thinking about geometry.

I had a box of cake mix, a can of frosting, and square and round pans. I was faced with a few design issues. My plan was to cut the round cake in two and attach the two halves to the square as the humps of the heart. So I had to make sure the side of the square pan was equal to the diameter of the round pan. That was easy. I chose 8-inch pans.



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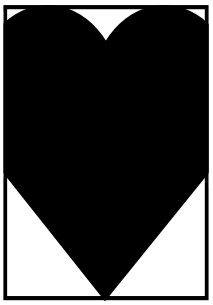
I asked students to speculate: "What do you think I did to make the cake this shape? What pan(s) do you imagine I used?"

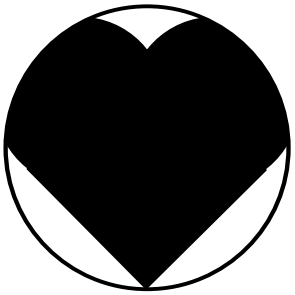
Student: "You used a square pan and cut out a heart."

(I made a mental note that she called the rectangle that she drew a "square"):



Teacher: "That's one way, but that's not how I did it."

Student: "I think you had a big circular pan and cut off pieces. Like this:"



Teacher: "That's another way, but not how I did it."

Student: "You had a heart-shaped pan."

Teacher: "No, but I'll give you a hint. I used two pans, a square one and a round one. Here they are."

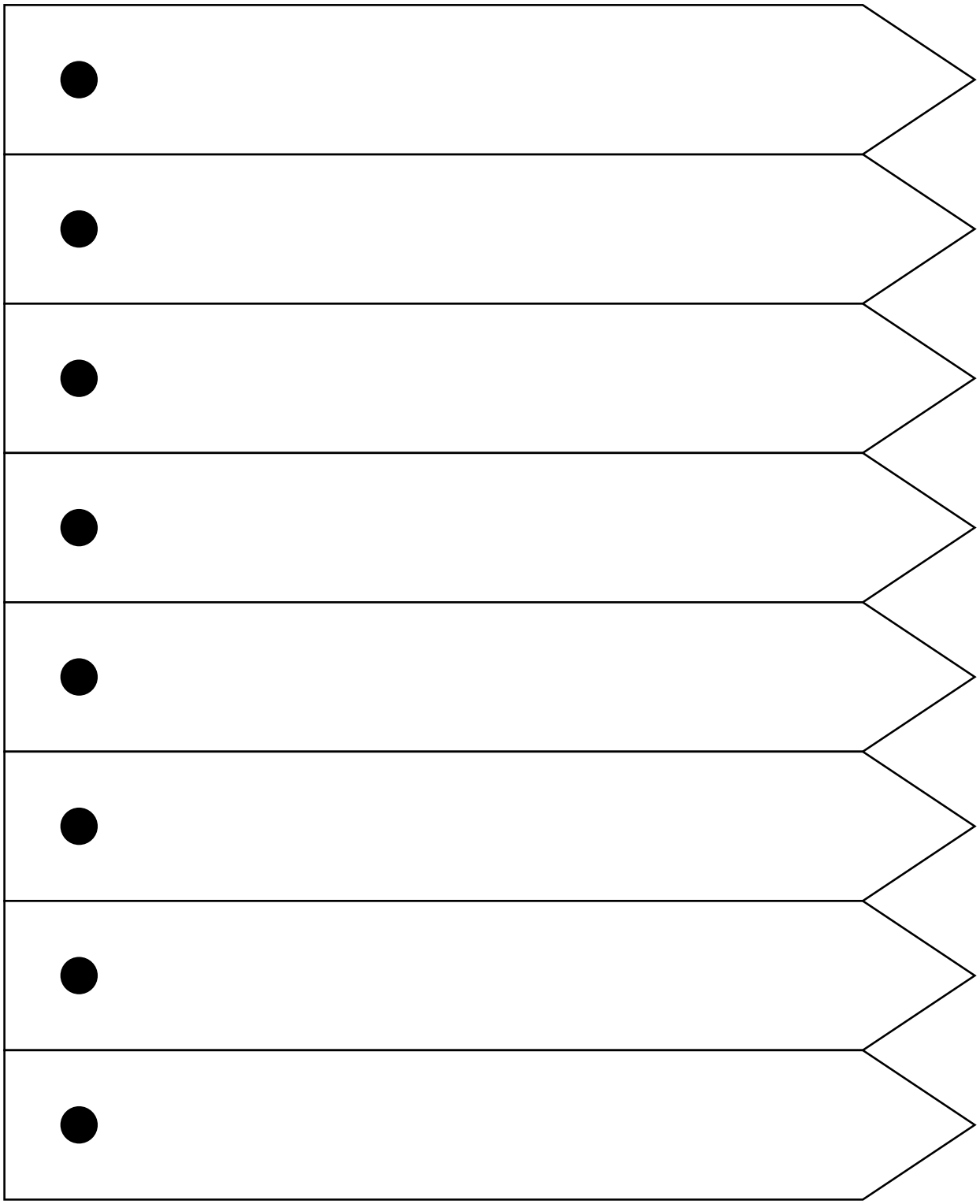
(Silence.) Then:

Student: "Okay, I see two parts of the circle, and you pushed the square between them." (He gestured over the cake.)

Teacher: "Yeah, that's exactly what I did. Who's hungry?"

Mary Jane Schmitt
Bridge to Learning and Literacy, Harvard University, MA

Blackline Master 1: Angle Demonstrators



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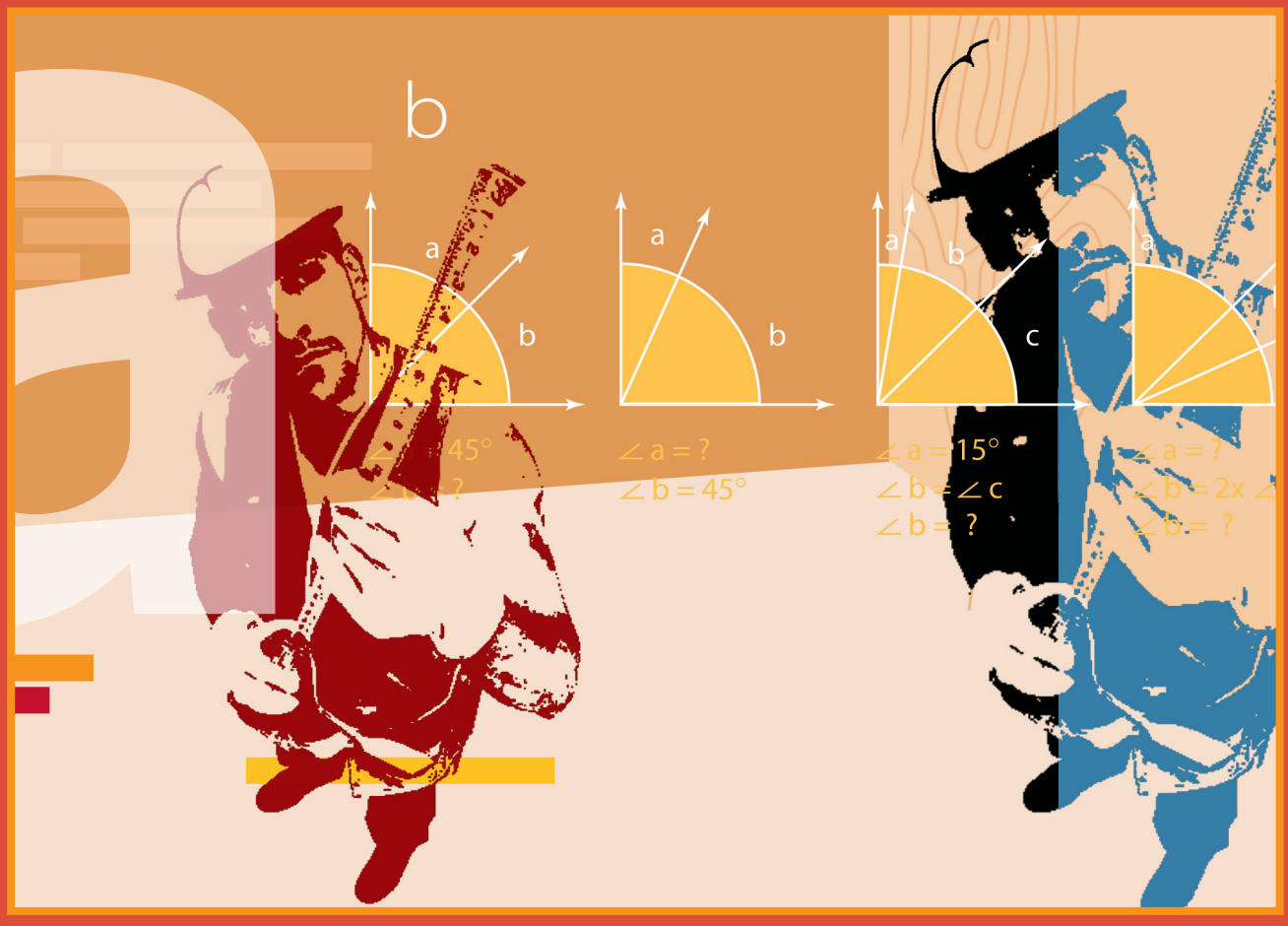
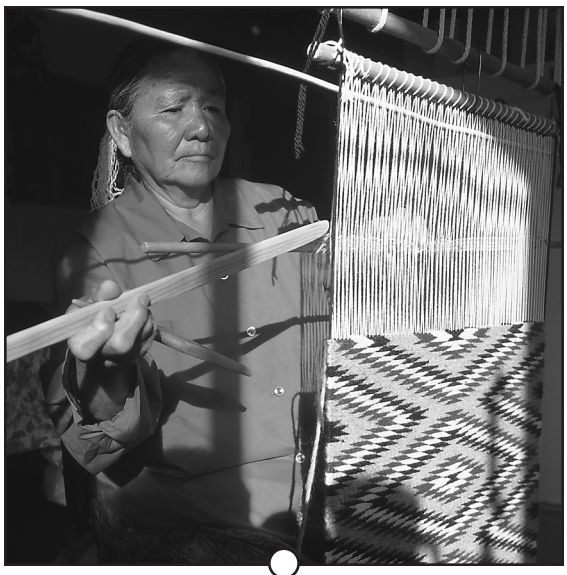
Geometry and Measurement



STUDENT BOOK



Opening the Unit: Geometry Groundwork

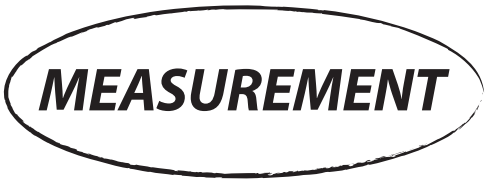
Is there geometry in your life?

Often when we are making something, ideas associated with the study of geometry arise. Do you see shapes within shapes? How do parts fit together? How does measuring help you figure things out when you are cooking, building, or sewing?

In this unit, you will learn about shape, line, and form. You will explore the skills that allow people to work mathematically with shapes in everyday life.

Activity 1: Making a Mind Map

Make a Mind Map using words, numbers, pictures, or ideas that come to mind when you think of *geometry* and *measurement*.



Activity 2: Making an Angle Demonstrator

Make your own angle demonstrator with the parts that your teacher will give you.

- Cut out the two strips of paper and place one on top of the other so the two dots are aligned.
- Put a fastener through the two dots so the paper strips can open and close like a mouth.

Now you have an angle demonstrator. Make another one for home use, if you like.

Activity 3: Initial Assessment

Your teacher will show you some problems and ask you to check off how you feel about your ability to solve each problem:

___ Can do ____ Don't know how ____ Not sure

Have you ever noticed that every new place you work has its own words or specialized vocabulary? This is true of topics in math too. In every lesson you will be introduced to some specialized vocabulary. Do not worry if you see words in the problems that you do not recognize. You can write some words down and look them up later, or learn as you go.

Activity 4: Homemade Objects

Choose an object from the collection provided, or use a homemade object of your own.

Object: _____

1. Sketch your object.

Activity 4: Homemade Objects *(continued)*

2. What shapes make up your object? Make a sketch of each shape you see. Name each shape.

Sketch Sheet

Sketch of Shapes Observed	Name or Description of Shapes

3. When you finish drawing and naming the shapes, trade your object and Sketch Sheet with a classmate. See if you can find any additional shapes in your partner's object. Add them to your partner's Sketch Sheet and name them.

Practice: Using Geometry

Write three paragraphs about how you use geometry and measurement.

- In your work…

- At home…

- In everyday life in your community…

Practice: Seeing Geometry

1. Sketch an object that has some interesting shapes.

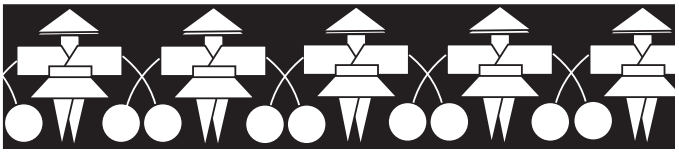
2. What shapes and angles do you see in the object?

3. What measurements could you take of the object?

Extension: Name the Shapes

The following are traditional Japanese patterns.

1. Sketch and name the shapes you see within this pattern.



2. Sketch and name the shapes you see within this pattern.

3. Many countries have traditional designs that include geometric shapes.

a. Research a design from your own or another tradition.

b. Sketch the design.

c. Name the shapes you see within the design.

Geometry and Measurement Unit Goals

- Recognize and describe shapes and their characteristics.
- Find area and perimeter of rectangles and volume of rectangular solids.
- Make drawings to scale.
- Use linear, square, and cubic units.
- Use spatial reasoning to solve problems.
- Make generalizations about two- and three-dimensional shapes.

My Own Goals