The Mathematical Learning of Zoombinis

Zoombinis is an engaging game, but it is also a mathematical learning environment. As the player explores the program, he or she will discover that it is much more than a collection of activities. Its 12 puzzles, each with four levels, work together to introduce important mathematical skills and concepts. The following sections describe the mathematics of *Zoombinis* and provide suggestions for enriching and extending what the player learns at the computer.

Mathematics of Zoombinis

The mathematics of *Zoombinis* is the math of the information age. It is the math used in writing computer programs, solving complex problems, organizing data in spreadsheets and searching for information on computer networks. *Zoombinis* offers an opportunity to learn mathematical thinking skills, as well as mathematical content.

Mathematical Thinking

Helping develop mathematical thinking skills is the primary focus of *Zoombinis*. The program doesn't tell the player what to do – it gives them the freedom to figure out each problem or puzzle and invent a solution. This process of solving the puzzles is the process of mathematical thinking: organizing information in relevant ways, reasoning about evidence, and testing in a systematic manner.

Organizing Information

In gameplay, this includes noting what the goal is, how many Zoombinis are present, and how each individual Zoombini might figure into the solution. Arranging Zoombinis by common traits or combinations of traits are two examples of how organization can help in the process of solving the puzzles.

Reasoning About Evidence

This is an essential part of gameplay. In most of the puzzles, the player will make a number of unsuccessful attempts; however, these failed attempts provide the very evidence needed to solve the puzzle: new insights are gained, options can be eliminated and others identified as still possible. In gameplay, as in life, as in formal, mathematical problem solving, valuable information can be obtained from all efforts, whether or not the efforts result in immediate success.

Systematic Testing

Developing smart trial-and-error skills is often essential in gameplay. In some of the puzzles there are a limited number of tries. Those players who are efficient in developing theories (or hunches) and testing these theories in an efficient manner are rewarded with the most success.

Mathematical Content

The mathematical content of *Zoombini*s includes logical thinking, comparing, grouping, graphing, sorting, and algebraic thinking. These ideas range from the most basic principles of logical thinking (cause and effect, order and sequence) to concepts that are studied formally in college.

Much of the mathematical content of the program is based on the Zoombini characters' *attributes* or *features* (variations in hair, eyes, nose, and feet), as well as attributes of other characters and props in the program. When the player works with attributes, he or she is working with variables or quantities of data–the building blocks of mathematical equations. Knowing how to solve for variables and combine them to form equations are crucial skills in mathematics. The *Zoombinis* gameplay provides an innovative way of gaining experience with attributes in the problem-solving process.

The puzzles in *Zoombinis* are organized along four paths. Each path contains puzzles that develop related mathematical ideas.
Trail Name Mathematical Focus
The Big, the Bad, and the Hungry Trail Theory testing: sorting, grouping, and comparing information.
Who's Bayou? Logical relationships: ordering, linking information, and finding patterns
The Deep, Dark Forest Trail Organization: graphing and mapping information in a variety of ways.
The Mountains of Despair Sorting, organizing, and algebraic thinking

Educational Benefits & Puzzle-Solving Strategies

Zoombini Isle

Zoombini Isle is the place where the player selects the attributes for each Zoombini before beginning the journey. By constructing Zoombinis, children become acquainted with features or attributes. The 20 features charted at the bottom-left of the computer screen on the Zoombini Picker are grouped into four attributes: hair, eyes, nose color, and feet. Each Zoombini has exactly one feature from each attribute. This structure forms the basis for most of the puzzles in the game.

Children can experiment by making Zoombinis with different features. For instance, groups can be made in which the features are very evenly distributed, or with one or two common features, or groups can include one or two pairs of identical or very similar Zoombinis. Sometimes having similar groups makes it a bit easier to navigate the puzzles, while more variation increases the challenge.

The Big,The Bad, And The Hungry Trail

The three puzzles that make up this trail are all "guess my rule" games. The Allergic Cliffs, the Cave Guards, and the Pizza Trolls each have a hidden requirement that the player must deduce. Many players tackle the puzzles intuitively at first, and gradually develop theories to explain and describe their strategy. As they learn to solve these puzzles, children deepen their understanding of the relationships among attributes, features, and sets.

The Allergic Cliffs

- observation

- forming and testing theories
- forming sets
- using evidence
- logical reasoning

When the player figures out that "only the red-nosed ones can go over this bridge," she or he is using the fundamental logical principle that a group of objects can be defined by a common feature (such as red noses). When the player adds that "only the ones that don't have red noses can go over this bridge," he or she is describing a more complicated kind of set, defined by *not* having certain features. Defining sets by what can or cannot be included is an important foundational principle in mathematics.

In addition to the mathematical content of sets and grouping, this puzzle emphasizes the mathematical process of forming theories (guesses based on evidence, predictions) and comparing them carefully to the evidence. For example, in this particular puzzle there are many possible explanations for the cliffs' behavior; as the game progresses the player can rule out more and more possibilities. Levels Very Hard and Very, Very Hard of this puzzle have more complicated kinds of rules, and challenge the player to consider a greater number of features. Because there is less room for error, they also demand careful reasoning in order to make the most of every bit of evidence.

Stone Cold Caves

- observation
- forming and testing theories
- forming sets
- using evidence
- logical reasoning

This puzzle is similar to Allergic Cliffs, but an extra twist is introduced. Now there are two pairs of characters (the bigger rocks in the center and the smaller rocks on either side), each with its own independent way of sorting the Zoombinis.

In this puzzle, a particular feature is important to the big rocks and a different feature is important to the small rocks. In addition, each member of a pair of rocks will reject a feature that the other likes. To master the Not So Easy level, simply learning good trial-and-error skills (making sure that a Zoombini doesn't get rejected twice by the same rock) is a major first step. Eventually, players learn how to use the rejection clues to sort out the separate groupings being applied by the big rocks and the small rocks. As in other puzzles, recognizing what one character rejects can help reveal what the opposite character will accept.

At the higher levels, players must consider a greater number of features used by the guardians to sort the Zoombinis. This requires that a player carefully examine Zoombinis that have successfully passed to determine the guardians' sorting rules.

Pizza Pass

- observation

- logical reasoning
- collecting evidence
- forming and testing theories

Pizza toppings represent yes/no choices which can exist in any combination, unlike the features of Zoombinis (you can't have a Zoombini with a red nose and a green nose). Because of this difference, the Pizza Pass puzzle requires different reasoning than the two preceding puzzles. In this case, there is only one combination of "features" (toppings) that will yield a solution.

This puzzle also requires a strategy for collecting evidence. Simply "going for the answer" is not always effective: the player must be careful to make pizzas that will yield good information even if they are not the final answer. Strategizing for evidence is an important problem-solving skill, and thinking about the information value of "wrong" answers will be very useful in forming a theory about which pizza the Tree Troll wants.

At the higher levels, there are more Tree Trolls to feed and the player must also make sundaes. Players must deduce which food item the trolls are responding to when they reject an offering. Paying close attention to what one troll dislikes can help reveal what another troll likes.

Who's Bayou? — the Northern Trek

The puzzles on this trail have to do with arranging objects by similar features. Players are challenged to recognize similarities that can develop or reveal patterns.

Captain Cajun's Ferryboat

- arranging by attributes
- logical reasoning
- comparing
- observation

Comparing—looking for similarities between Zoombinis—is the basic skill involved in this puzzle. Players understand that Zoombinis can be similar in several ways (hair, eyes, nose, or feet), and use this information to organize the Zoombinis in the boat.

Random trial and error may work at the Not So Easy level, but as the player moves up through the levels, an arrangement that fits all Zoombinis on the boat becomes more difficult to find. It becomes increasingly important to be systematic (making the most of every bit of evidence), in order to find a solution. Not every mathematical problem has a straightforward computational solution, and systematic trial-and-error steps are essential thinking tools used by mathematicians and scientists.

In addition, paying attention to common and uncommon features in a band of Zoombinis is a good example of the statistical thinking that is useful in many of the puzzles. Look for a few very common features among your group of Zoombinis and cluster the Zoombinis in the boat according to these features. Then, use other kinds of similarities along the borders of the clusters.

Titanic Tattooed Toads

- observation
- comparing
- finding and creating patterns
- predicting

In this puzzle, the similarities are between lily pads, not Zoombinis. As players look for paths through the field of lily pads, they selectively pay attention to only one attribute at a time–a good introduction to pattern recognition. Careful scrutiny will reveal several different patterns based either on shape or color.

In levels Oh So Hard through Very, Very Hard, the player can make paths by swapping lily pads. But how can you make new paths without destroying old ones? How can you make paths efficiently to avoid running out of swaps? What kinds of paths can cross and what kinds can't? These problems intensify at higher levels, and logical thinking and theory testing are needed to solve them.

Stone Rise

- observation
- sorting by attribute
- comparing
- creating networks or logical groups

In order to get past this obstacle, the Zoombinis must link up in ways that complete "electrical" circuits. As in the Ferryboat puzzle, the Stone Rise puzzle involves looking for similarities between Zoombinis. But where the Ferryboat puzzle can be solved using any similarities, the Stone Rise puzzle requires that specific similarities be met in specific places. The Not So Easy level can often be completed using simple trial and error, but the higher levels are more easily solved if the player plans ahead and groups Zoombinis by common traits before beginning the puzzle. A typical strategic question for the player might be whether to first link Zoombinis with the most common traits or those with uncommon traits.

In addition, the Very Hard and Very, Very Hard levels introduce the concept of network connections. While two Zoombinis may match according to the attribute shown on a stone pad, they must be linked to others along a path of stones and connect to the power source. In addition, Zoombinis that aren't adjacent might nevertheless be connected. What matters is not the physical distance between two Zoombinis, but rather the shortest number of connections required to link them up. This concept is important in everything from the design of computer networks to the scheduling of airline flights.

The Deep, Dark Forest Trail — the Southern Trek

Each attribute of the Zoombinis (hair, eyes, nose color, and feet) is a mathematical dimension. The puzzles on this path are dimensional arrangement puzzles where an attribute of the Zoombinis is mapped onto a spatial dimension. These kinds of arrangements are a foundation of graphing and other forms of data representations.

Fleens!

- observation
- functions and mapping
- collecting evidence
- forming and testing theories

To lure the Fleens off the tree branch, the player must figure out the relationships between Zoombini features and Fleen features. This process of finding relationships or corresponding aspects of one group (Zoombinis) to another (Fleens) is also called a function or mapping. Functions and mappings are important ideas in algebra.

At higher levels, the relationships become more complex. For example, instead of Zoombinis' noses being directly related to Fleen noses, now the relationship may be between Zoombini noses and Fleen feet. Looking at the distribution of features in one's band of Zoombinis and relating this to the distribution of Fleen features can yield valuable information.

Hotel Dimensia

- observation
- grouping and organizing
- considering multidimensional arrangements of data
- mapping
- forming and testing theories

The Zoombinis need to stay in the hotel for the night, but only certain ones can stay in particular rooms. The hotel introduces mapping or arranging data dimensionally. Certain features can be arranged along a dimension: all ponytails on the first floor, or all sunglasses in the left-most column, for instance. At the Not So Easy level there is only one dimension. Oh So Hard and Very Hard involve two dimensions, and the Very, Very Hard level introduces a system for mapping three dimensions on a two-dimensional screen.

In order to successfully complete the puzzle, the player must also have a strategy for testing theories. For example, players may consider which Zoombinis they will place in order to test whether or not a certain feature is arrayed on the horizontal dimension. In addition, the closed rooms in the Very Hard level require the player to consider the distribution of features in the current band of Zoombinis before testing any theories.

Mudball Wall

- observation
- functions and mapping
- considering multidimensional arrangements of data
- graphing
- forming and testing theories
- finding patterns

To get the Zoombinis over the wall the player must select the mudballs that will splat against the correct targets. At the Not So Easy level the puzzle offers a variation on the dimensional mapping present in the Hotel Dimensia puzzle. But beginning with the next level, permutations

are also introduced. Permutations are consistent patterns of transformation which will become apparent as you watch mudballs strike the wall. In this puzzle they are visual representations of sophisticated algebraic concepts. Also, at the Very Hard and Very Very Hard levels, the player must now consider three variables that will determine where a mudball will strike.

The Mountains of Despair – the Subterranean Journey

The puzzles along this path involve sorting, organizing, and algebraic thinking. The player will not only consider appropriate arrangements of Zoombinis by attributes but also the way in which new variables can change or affect them.

The Lion's Lair

- observation
- sorting and ordering
- organizing data
- forming and testing theories

The Zoombinis must line up on the path according to a sort order that is represented by clues on the wall of the cavern. Sorting is important for understanding concepts like alphabetical order, or the arrangement of data in a database. In order to deduce the sort order, the player must look at the distribution of the Zoombinis' traits, and form theories that can then be tested. Thinking strategically about the similarities between the Zoombinis will help the player avoid too many wrong guesses.

Although numbers never figure explicitly into *Zoombinis*, this is an example of a puzzle in which numerical reasoning is useful. For instance, the player can sometimes find the solution by counting the number of empty stones on a section of the path, and relating that to the number of Zoombinis present with a certain trait.

Beginning with the Oh So Hard level, there are fewer clues shown on the wall, challenging the player to deduce the order through observation and testing. At the Very Hard and Very, Very Hard levels, the concept of primary and secondary sorting is introduced as players need to group Zoombinis according to two traits within these groups. Once Zoombinis are grouped by their primary attribute, they are further ordered according to the secondary attribute.

Mirror Machine

- observation
- matching
- organizing data
- algebraic thinking

At the Not So Easy level this puzzle is a simple matching task. Players will find that they can make the task easier by arranging their Zoombinis in an organized way, for example, by placing all of the Zoombinis with orange noses in the same area

At the upper levels, the puzzle becomes a challenge in algebraic thinking. Algebra is more than just solving for x and y. In this puzzle, the crystal filter plates that change the Zoombinis'

features represent the important algebraic idea of a transformation, where functions of new variables replace the original variables.

At these levels, the puzzle is arranged in a kind of "Zoombini equation." The central mirror is like an equals sign, the transformer crystals are like functions, and the slots for Zoombinis are like x and y variables! In this way, the player's challenge is to balance both sides of the equation so they are equal.

Bubblewonder Abyss

- observation
- sequencing
- predicting
- organizing information

For their final challenge the Zoombinis must cross the abyss by moving across a grid. Along the way, they must pass through a series of devices shown as symbols on the grid which will either trap them or continue them on their way. In order to successfully navigate the Abyss, the player must learn to recognize certain symbols in the puzzle and predict their behavior. It is particularly important to understand the way different symbols interact when encountered in different sequences. Singly or in sequence, these devices are similar to the algorithms a computer programmer uses in constructing software. In learning to think systematically about these interactions, the player is engaged in the same process of logical reasoning as a software engineer.

As the difficulty level increases, so does the complexity and number of these algorithms, but even the most complicated Very, Very Hard level puzzle behaves in consistent and predictable ways that players can learn to deduce. As in many other puzzles in this game, success also requires that the player learns to look analytically at the distribution of traits present in every band of Zoombinis. Relating this to the layout of devices on the grid helps the player plan the correct sequence to get all of the Zoombinis across.