UNIT 14: LESSON 1

Designing with Cylinders

OVERVIEW	
Unit Title: Exploring Cylinders with Product Design	**Length of Lesson in # of Hours:** 3 **# of Classes:** 1
How does this lesson connect to previous or future work as exemplified by the Standards in your scope and sequence? This lesson combines prior exploration of circle and rectangle properties and connects those shapes to cylinders and properties related to them for finding surface area.	

LESSON OBJECTIVES

At the end of this lesson, students will be able to:

- recognize and use 2D representations of 3D objects
- connect and apply properties of circles and rectangles to properties of cylinders

STANDARDS

Citation	*[*This portion of the standard will not be explicitly covered in this lesson.]*
6.G.4	Represent three-dimensional figures using nets made up of rectangles *[*and triangles]*, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
7.G.4	Know the formulas for the area and circumference of a circle and use them to solve problems; [*give an informal derivation of the relationship between the circumference and area of a circle.]
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive [*and negative] rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *[For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 ¾ inches long in the center of a door that is 27 ½ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.]*

1 - 3 MATHEMATICAL PRACTICE(S) ADDRESSED IN THIS LESSON	ELEMENTS OF RIGOR
MP 1: Make sense of problems and persevere in solving them. MP 2: Reason abstractly and quantitatively. MP 3: Construct viable arguments and critique the reasoning of others. MP 4: Model with mathematics. MP 6: Attend to precision.	*Which aspect(s) of Rigor do the targeted Standard(s) require?* ☒ Conceptual understanding of key concepts ☐ Procedural skill and fluency ☒ Rigorous application of mathematics in real-world contexts

ESSENTIAL QUESTIONS

How can knowing about circles and rectangles help to understand the formula for finding the surface area of a cylinder?

Where does each part of the surface area of a cylinder formula show up on the solid figure?

EVIDENCE OF LEARNING

Ways I and my students will know the extent to which the objectives have been met.

Students will be able to apply their understanding of circumference and area of circles, as well as area of rectangles, to finding the surface area of cylinders. Students also will know when not to use certain portions of a formula.

LEARNING PLAN - Vocabulary

cylinder

LEARNING PLAN - Introduction	MATERIALS	TIME
Creating a cylinder		
1. Ask students if they know what a **cylinder** is and to give a description and examples. If no students recognize the term, give an example of a tin can and then ask students if they can		

describe what that shape is and give other examples. Ask them if they can find any cylinders in the room (if there are any). Arrive at a working definition of a cylinder.		
2. Distribute paper, scissors, and tape and ask students to attempt to construct a cylinder. Do not give them any hints or guidance. ***Note to teacher**: This is a challenging task and students will likely encounter problems. The purpose of the activity is to have students realize two things: 1) that the shape of the paper that makes the wall of the cylinder is a rectangle, and 2) that the width of the rectangle has to match the circumference of the circles. It will be very hard to get the width of the rectangle to match the circumference of the circle without measuring and calculating, so stop students before they get frustrated and tell them that the point of the exercise was to figure out what the challenges are in creating a cylinder and what shapes the pieces are.*	Blank paper Scissors Tape	
3. Debrief the challenges involved in creating a cylinder and what students have learned about what is involved. Make sure students arrive at the two important points above.		
LEARNING PLAN – Body of the Lesson	**MATERIALS**	**TIME**
Introducing the task		
1. Follow the write-up for the Mathematics Assessment Project (MAP)'s *Designing a 3D Product in 2D: A Sports Bag*, *Introducing the task: Designing a Sports Bag* (p. T2-T3) to evaluate if students can recognize and use common 2D representations of 3D objects. ***Note to teacher:** The write-up of this task at MAP states that it is intended to be completed in advance of the class, but for the purpose of this lesson, students should do the task in class. Also, it is highly recommended that you use the slides from the projector resource provided with the link above as you navigate this activity. In it you will find visual models of the work you're explaining, as well as step-by-step instructions of the assigned activities.*	*Designing a Sports Bag* (p. S1) [one copy per student]; http://map.mathshell.org/lessons.php?unit=7305&collection=8 Blank paper *Formula Sheet* (can reference either the GED® or HiSET® formula sheet given out in earlier lessons) [if needed] Calculators Rulers Graph paper	

2. Have students work with a partner in class on the problem. Give students about 25 minutes to wrestle with the problem on their own before moving on. Even if students haven't completed all of the questions, make sure they have attempted all questions on the page. Take note of student approaches and any problems that arise. Watch for some of the *Common Issues* (shown on p. T4). ***Note to teacher**: You may notice that in the write-up of this activity at MAP it says to have students work individually on this task. This is because the activity was originally designed as an assessment, but in this context you are using it as a discovery activity. Because CALM is strongly focused on group work, you are encouraged to have students work with partners, but if it fits your class better to have them work individually, you can make that choice.*		
3. Without going over the solutions, discuss any issues that might have come up as you walked around the room and observed students working. Revisit any of the *Common Issues* you observed. Use any of the *Suggested questions and prompts* (p. T4) that apply. The purpose is to get students to engage the problem-solving process as a group. ***Note to teacher:** Remind students that activities like we are doing today are about problem-solving, and there are several approaches that likely will be taken by your classmates. Be sure to share your ideas even if you aren't sure you found a good estimate. All answers are helpful for this activity.*		
Collaborative activities: comparing and analyzing approaches		
4. Follow the instructions for *Designing a 3D Product in 2D: A Sports Bag*, *Collaborative activity: making posters* (p. T5). ***Note to teacher**: Depending on how students are doing with this task, you may want to reorganize groups as is recommended in the write-up at MAP or keep students with their partners for making posters. If you keep them with their current partners, then be sure to have each group present so that students can see and talk about different approaches to the task. If you reorganize the groups, split up partners and attempt to group students who have used different strategies and push them to combine their approaches. In that case, you may want to do a gallery walk before coming together to discuss the posters.*	Chart paper Markers	
5. If you reorganized the groups so that students had to combine their approaches, follow the instructions for *Designing a 3D Product in 2D: A Sports Bag*, *Sharing different approaches* (p. T6) to process the groups' methods for producing a solution as a team.		

6. Follow the instructions for *Designing a 3D Product in 2D: A Sports Bag*, *Collaborative analysis of Sample Responses to Discuss* (pp. T6-T7) to have students compare their work with the solutions provided in the activity. Solutions on pp. T6-T7 do not need to be revealed to students; however, they may be of use as you offer guidance to students struggling to understand any solutions. ***Note to teacher (from p. T6):*** *"It may not be appropriate, or there may not be enough time, for all students to analyze all three sample responses. Each response highlights different misconceptions and so, depending on the progress already made on the task, it may be appropriate to issue different sample responses to different groups. For example, groups that have omitted some of the seam allowances could be given Ben's work, while groups that have rounded to an inappropriate degree of accuracy or omitted to check for alternative arrangements could be given Aisha's work."*	*Sample responses to discuss* (pp. S3-S5) [one set per small group]	
Whole-class discussion, small-group work, and follow-up lesson		
7. Follow the instructions for *Designing a 3D Product in 2D: A Sports Bag*, *Whole-class discussion: comparing different approaches* (pp. T7-T8) to talk about the different approaches used in the sample work, as well as by the students. Solutions are shown on pp. T9-T10.		
The surface area formula		
8. Remind students that when studying rectangular solids in the last unit, they investigated both surface area and volume. Ask them which of those topics is connected to the work they just did with the sports bag. [Surface area]		
9. Ask students to tell you what the components are of the surface area of a cylinder. If necessary, remind them that with the rectangular solid they were concerned with the top, bottom, front, back, and left and right sides. Here, they should recognize that the parts are the two circles and the rectangle forming the tube that connects them. (The technical name for the area of that rectangle is the **lateral area** of the cylinder, but that vocabulary is not necessary to bring up unless students want to know the name for it.)		
10. Ask students to try on their own to come up with a formula for the surface area of a cylinder, using r to represent the radius of the cylinder and h to represent the height.		
11. Have them compare their formulas with a partner and see if they agree.		

12. Ask students to share their formulas and record them on the board. Collect as many variations of the formula as students have and accept any that are incorrect as well. 13. Discuss the formulas as a group, identifying the parts in each one. For example, in the formula $SA = 2\pi r^2 + 2\pi rh$ the first term represents the two circles and the second term represents the rectangle. In the formula $SA = \pi dh + \pi r^2 + \pi r^2$ the first term represents the rectangle and the second and third terms represent the two circles. Be sure to identify the three parts in each formula and make the connection between the length of the rectangle and the circumference of the circles. If any formulas are incorrect, discuss why they are incorrect. For example, a student might come up with only $2\pi rh$, forgetting about the two circles. 14. Check to make sure every student understands the correct formulas. ***Note to teacher**: It's possible that students will use πd instead of 2πr to represent circumference in the formula. This is technically fine, but they should know that it is conventional to write formulas with fewer variables where possible so using 2πr is preferred.*		
LEARNING PLAN – Closure / Conclusion	**MATERIALS**	**TIME**
Exit ticket prompt: What have you learned about cylinders?	Exit ticket handout U14.L1	

ADDITIONAL PRACTICE	**MATERIALS**
For further exploration with the surface area of cylinders, students can play around with the interactive tool at the bottom of the website page. The description provided above the interactive also serves as a nice review of what was covered in this lesson.	Annenberg Learner Interactives: Geometry 3D Shapes - Cylinder https://www.learner.org/series/interactive-geometry-3d-shapes/
Students can use this virtual cylinder to explore what happens to the surface area when the base and height are adjusted on a cylinder. Additional vocabulary and explanation, as well as practice problems are provided below the interactive tool.	Math Open Reference: Surface Area of a Cylinder http://www.mathopenref.com/cylinderarea.html